

1959

Basic column strength. Original manuscript for "Basic Column Strength," September 1959 and publications Proc. ASCE, 86 (ST7), p. 139, (July 1960)

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RESIDUAL STRESS AND THE COMPRESSIVE PROPERTIES OF STEEL

BASIC COLUMN STRENGTH

by

Lynn S. Beedle and Lambert Tall

This work has been carried out as a part of an investigation sponsored jointly by the Column Research Council, the Pennsylvania Department of Highways, the Bureau of Public Roads, and the National Science Foundation

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September 1959

Fritz Laboratory Report No. 220A.34

DEPARTMENT OF CIVIL ENGINEERING
FRITZ ENGINEERING LABORATORY
LEHIGH UNIVERSITY
BETHLEHEM, PENNSYLVANIA

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ABSTRACT

A summary is presented of a theoretical and experimental investigation of the strength of centrally-loaded columns as influenced by residual stresses and variations in the yield stress level. It is shown that the basic strength of structural steel columns containing residual stresses may be expressed in terms of the tangent modulus. Approximations suitable for design use are suggested. Information on both the yield stress level and the magnitude and distribution of residual stress in structural members is presented.

1. INTRODUCTION

A research project on the "Influence of Residual Stress on Column Strength and the Mechanical Properties of Rolled Shapes" has been in progress at Lehigh University under the guidance of Research Committee A of the Column Research Council. This Committee was assigned the task of determining the relationship between material properties and the strength of columns, and the first pronouncement of the Council (based on the recommendation of Committee A) was its Technical Memorandum No. 1 entitled "THE BASIC COLUMN FORMULA".⁽¹⁾ This memorandum states that the critical or ultimate failure load of a column is given by the equation

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} \quad \dots (1)$$

This formula cannot be applied to steel columns if the stress-strain relationship is determined from a small coupon cut from the section. Early work clearly showed this to be true, and in later studies completed prior to the time that this general investigation was started it was shown that residual stresses might account for differences in column

strength of as much as 30% below that which would be inferred from coupon tests.

The column curve depends upon the stress-strain relationship. The latter, in turn, is dependent upon two important factors; these are: (a) the magnitude and distribution of residual stresses, and (b) the basic yield stress level of the material. Therefore, the objectives of the investigation were: (1) to determine the magnitude and distribution of residual stresses in columns, and (2) to develop methods of predicting the influence of these residual stresses on column strength. As a necessary parallel study, the program included a determination of the basic yield stress level of the material of which columns would be fabricated.

The program included tests of coupons of the type performed in the mill, tests of stub columns (short lengths of full cross sectional area), and column tests. For the same shapes, residual stresses were measured by the sectioning technique. Theories were developed for predicting column strength, and from the measurements made it was possible to obtain a correlation with the theory. Maximum strength column formulas could then be written.

It is the purpose of this report to summarize the findings of the investigation to date and to discuss the significance thereof. Reference is made, throughout, to the various progress reports that contain the detailed experimental and theoretical work. This paper is concerned primarily with rolled wide-flange shapes of ASTM Designation A7 structural steel. A brief discussion of built-up columns (welded and riveted) and of low-alloy high-strength steel columns is included. The scope is limited to centrally-loaded columns.

2. RESIDUAL STRESSES

(1) Formation of Residual Stresses

Residual stresses are formed in a structural member as a result of plastic deformations. In rolled shapes these deformations always occur during the process of cooling from the rolling temperature to air temperature; the plastic deformations result from the fact that some parts of the shape cool much more rapidly than others causing inelastic deformations in the slower cooling portions. (The flange tips of a WF shape, for example, would cool more rapidly than the juncture of flange and web.) The mechanism by which residual stresses are formed has been described in Refs. 2, 3, and 8.

Residual stresses also are formed as a result of fabrication operations. The process of cold-bending that is required in the straightening operation and the process of cambering both introduce residual stresses due to plastic deformation.^(3,8) Residual stresses are also introduced during the welding operation as a result of the localized heat input and resultant plastic deformation.⁽⁴⁾

Thus, plastic deformations are necessary in order that residual stresses be formed. In hot-rolled or welded members, the part to cool last is usually in a state of tensile residual stress.

(2) Magnitude and Distribution of Residual Stresses

Methods for determining cooling residual stresses in plates and for obtaining a qualitative estimate of stresses in WF shapes are available.⁽³⁾ The magnitude and distribution of these stresses depend on the shape of cross section, initial temperature, cooling conditions, and material properties.

The measurement of residual stresses confirms the trends predicted theoretically. A considerable number of such

measurements have been made and they permit a good estimate to be made of the magnitude and distribution of residual stresses likely to be encountered in hot-rolled WF members.^(3,5) For the purpose of making these measurements shapes were selected to have widely differing geometry. Figs. 1a, 1b and 1c present some of the measured results, showing the magnitude and distribution of stresses across the flange and web. While the variation is considerable, the general pattern in the flange is similar. The residual stresses were determined by the "method of sectioning" described in Ref. 2.

The magnitude and distribution of residual stresses in the flanges also may be estimated from the results of a stub-column test. Refs. 2 and 11 describe these methods.

Table 1 contains a listing of all shapes studies thus far. It also contains a summary of all the measurements made in the program. Table 2 presents the average value of the residual stress at different positions in the cross section and gives the variation as well. Insofar as columns are concerned, the most important of the stresses are those at the flange tips (σ_{rc}) and from this program of tests the average compressive stress σ_{rc} is 12.8 ksi with a maximum

of 18.7 ksi and a minimum of 7.7 ksi.

The magnitude of the residual stress at the flange tips may also be determined indirectly from a stub-column test, and is the difference between the yield stress level and the proportional limit. This value has been found to be about 13.0 ksi which agrees well with the measured value of 12.8 ksi noted above. (3)

(3) Variation of Residual Stresses

In addition to average values, Table 2 gives the maximum and minimum values of measured residual stresses. Fig. 2a gives the frequency distribution of flange-tip stress as determined by actual sectioning. (3) Fig. 2b shows the same information as determined indirectly from the proportional limit obtained for the same group of "stub-column" tests. (3)

The variation of residual stress within material from one ingot is relatively small, but larger variations may exist between material from different lots. (2)

Theoretical studies show that cooling residual stresses are constant along the member except for a distance approximately equal to the larger cross sectional dimension at the

ends. This is due to the uniformity of the shape and its manufacture. Measurements are in agreement with the theory as shown in Fig. 3.⁽³⁾ While a trend may exist for the variation of residual stress as a function of the geometry of the cross sectional shape, it has not been possible as yet to show a precise relationship.

(4) Influence of Residual Stress on the Apparent Stress-Strain Relationship

As implied above, residual stresses affect the average stress-strain relationship of the complete cross section. Fig. 4 is presented to show this influence diagrammatically. Fig. 4a shows a short length of a wide-flange shape with a simplified linear distribution of residual stress in the flanges. The cutting of a coupon from the flange of the member would relieve the residual stresses that were present in the coupon prior to sectioning, and the stress-strain relationship determined from this coupon would be as shown by the dashed line in Fig. 4b. If, now, the load is considered as being applied to the entire cross section containing its residual stresses it is evident that when the applied stress becomes equal to the difference between

σ_y and σ_{rc} , then yielding will commence at the flange tips.

Thus,

$$\underline{\sigma_p = \sigma_y - \sigma_{rc}} \quad \dots (2)$$

The superposition of stresses at the "proportional limit" is shown in Fig. 4c. Yielding occurs when the flange tip residual stress σ_{rc} plus the applied stress σ_p is equal to σ_y .

When more load is applied, the average stress and average strain are no longer proportional to one another and a non-linear stress-strain relationship results for the section as a whole. Fig. 4d shows a WF shape with flanges partially yielded (shown shaded) and the corresponding stress distribution for $\sigma_{ave.} > \sigma_p$. Above the proportional limit and below the yield stress the strain is given by

$$\epsilon = \frac{1}{E} (\sigma_y - \sigma_{rx_o}) \quad \dots (3)$$

with σ_{rx_o} defined in Fig. 4d. (2) The average stress versus average strain curve for the entire cross section is shown in Fig. 4b, it being assumed that the shape is made up of two rectangles, and that the residual stress distribution

is linear with $\sigma_y = 40$ ksi, and $\sigma_p = 20$ ksi.

The yield stress level is unaffected by the residual stresses.

Thus, the effect of residual stress on the stress-strain relationship is to lower the proportional limit and to cause the stress-strain diagram to be non-linear beyond that point and up to the yield stress level. The proportional limit may be computed from Eq. (2). For a typical WF column ($A_f/A_w = 3.0$) with $\sigma_y = -34$ ksi and with $\sigma_{rc} = -13$ ksi the theoretical stress-strain curve is as shown in Fig. 5.

To obtain experimental correlation with the above predictions, tests and measurements of stub columns have been made. A typical stub column test is shown in Fig. 6. Typical results are shown in Fig. 7 which also shows the contrast with the curve based on coupon tests. Fig. 8 is the stress-strain curve of the annealed stub column (the residual stresses have been removed); the comparison with Fig. 7 clearly verifies the influence of residual stresses on the average stress-strain curve.

Of interest is the fact that the average stress-strain

curve, Fig. 5 is nearly identical with the average stress-strain curve determined from stub column tests, Fig. 18.

(5) Cold-Bending Residual Stresses

Residual stresses due to cold bending can be predicted with reasonable accuracy, ⁽³⁾ assuming certain initial cooling stresses and the extent of deformation. Some measurements are shown in Fig. 9. In general, the maximum and minimum stresses are of the same order of magnitude as the cooling residual stresses. While the influence on the stress-strain curve might be predicted, this has not been done as studies (described below) show it to be unnecessary.

3. MECHANICAL PROPERTIES

(1) Proportional Limit

The general effect of residual stresses on the stress-strain relationship was discussed in section 4 above. The proportional limit is reduced below that obtained in coupon tests, and may be computed either from Eq. 2 or may be measured in a stub column test. Fig. 7 clearly shows this effect, and further comparisons were made in Refs. 2 and 3.

The average value of the proportional limit is 21.7 ksi as determined indirectly by residual stress measurements in this program of tests. The average value determined by the offset method from stub column tests is 20.7 ksi, and the frequency distribution curve for these 40 specimens is shown in Fig. 10. Since the offset method was used to determine the proportional limit (Fig. 10 inset), the actual value is even lower than 20.7 ksi. It is to be expected that the stub column proportional limit would be lower than the value determined indirectly from Eq. 2. In the first place, the residual stresses are probably greater than the measured value by a small amount. Secondly, the flange-tip values of σ_{rc} in Table 1 are averages of measurements on the four corners and on two sides, and deviations from these averages will be reflected in a lowering of the proportional limit. Small inaccuracies of alignment also influence the observed proportional limit.

(2) Coupon Strength, Acceptance Tests, Strain Rate and the Yield Stress Level

The yield stress level may be determined from the results of an ASTM acceptance test or it may be determined from a

laboratory coupon test or from the results of a laboratory stub column test. Factors such as upper yield point, strain rate, and the web strength compared with flange strength cause the yield stress level of a full cross section (stub column) tested in the laboratory in compression to be markedly less than the tensile yield point determined in the routine ASTM acceptance test. (2,6)

Figure 11 illustrates the different stress-strain curves that may be obtained depending upon the type of test that is performed and shows diagrammatically the influence of the various factors as follows:

(1) Starting with curve A, the yield value reported in a mill-type acceptance test is usually (though not always) the upper yield point. Occasionally specimens will not exhibit an upper yield point (dashed line, curve B), in which case the yield stress is usually recorded at a strain of 0.5%. The upper yield point is from 0% to 10% higher than the yield stress level.

(2) Comparing curves A and C in Fig. 11, the ASTM acceptance test usually will show a higher yield

stress level primarily because of the effect of strain rate. The "static" yield level* of a web coupon is from 10% to 15% lower than the strength obtained in the acceptance test.⁽⁶⁾

(3) There is a difference in strength between the various elements of a rolled shape, the web being stronger than the flange. Thus the average yield level of a stub column is lowered still further in comparison with the mill acceptance test because the latter is made from the web (compare curves C and D).

(4) Finally, from Fig. 11 it is seen that a stub column (tested at "zero strain rate") reflects the effect of residual stress upon the stress-strain curve and averages out the differences between web and flange material, as shown by curve F.

Tests have shown that even a "very slow" laboratory strain rate used in testing coupons (an elastic strain rate of one micro-inch per inch per second) can raise the apparent yield stress level by as much as 5%.⁽⁷⁾ The effect of strain

*The static yield level is defined as the value measured at a zero strain rate.^(6,7)

rate on the yield stress level in a typical tension coupon test is shown in Fig. 12. From the results of such tests the relationship shown in Fig. 13 may be obtained. The measured yield level compared with the static yield level is shown as a function of the plastic strain rate. Since the plastic strain rate in a mill test is about 1000 micro-in/in/sec., increases in the yield stress above the static value may be expected to be as high as 16%.

It is of interest to note that the yield stress level obtained from a stub column test agrees well with tests of tension and compression coupons if the results of the latter are averaged according to their respective areas in the cross section, and if both the coupons and the stub column are tested at the zero strain rate.^(6,7) This is shown diagrammatically in Fig. 11 (compare curves E and F).

A comparison of the results of acceptance tests and of stub column tests is important in establishing a value for the yield stress to use in a basic column formula. If this relationship can be established for a fairly large sample, then it could be applied with confidence to the larger body of acceptance test data available in the mills to give a

reasonable estimate of the actual strength of structural steel columns. Fig. 14, 15 and 16 show the distribution of the yield stress as determined by a number of methods as follows: (2,6,7)

- | | |
|---------------------------------------|-------------------------|
| (1) ASTM acceptance tests in the mill | Fig.14 (3124 Specimens) |
| | Fig.15 (3010 Specimens) |
| | Fig.16a (35 Specimens) |
| (2) Simulated ASTM tests | Fig.16b (35 Specimens) |
| (3) Stub-column tests | Fig.16c (35 Specimens) |
| (4) Stub-column tests | Fig.16d (47 Specimens) |

Items (1), (2) and (3) (Fig. 16) are for the same control group. Comparing Fig. 16a with Fig. 16b shows that the mill test results may be approximated within about 4% by a laboratory test that simulates mill test procedures. The average value of the control group of mill tests (Fig. 16a) was 42.9 ksi with a standard deviation s of 4.4 ksi. Further comparisons of this type are shown in Refs. 6 and 7.

The average value of the yield stress level determined in simulated mill tests was 41.2 ksi (Fig. 16b), while the average strength of the corresponding stub columns (Fig. 16c)

was 34.0 ksi. The average value of the yield stress level for all stub columns tested in the program was 34.5 ksi with variations from 24.6 ksi to 43.0 ksi (Fig. 16d).

The probable ratio of the basic compressive strength (the static yield stress level) to the "acceptance test" strength was found to be 0.80. In Fig. 17 is plotted the distribution of the individual ratios of stub column yield level to mill test "yield point". The average is 0.80 with a minimum of 0.62 and a maximum of 0.92. From this information it would be concluded that the static yield stress level of a wide-flange column averages about 20% less than the value obtained in the ASTM-type acceptance test. As pointed out, this difference is due mainly to the strain-rate effect, but also is influenced by the higher yield stress of the web and the difference between upper and lower yield points.

Applying this average ratio (0.80) to the average value of the mill tests shown in Figs. 14 and 15 (41.7 ksi) there is obtained a probable compressive strength for this material of 33.4 ksi. The average value obtained from all stub columns tested in the program was 34.5 ksi suggesting

that the sample was fairly representative.

→ 4. CENTRALLY LOADED COLUMNS

1. Influence of Residual Stress on Column Strength

For a column with material exhibiting an idealized stress-strain curve and free from residual stresses, the buckling strength is defined by the Euler buckling curve and the yield stress of the material. [The existence of residual stresses in the cross section reduces the buckling strength, since there is an early localized yielding at certain portions of the cross section. This reduction is greatest when the slenderness ratio is between 70 and 90.

This effect of residual stress may be illustrated by an example. Consider a column of rectangular cross section containing residual stresses, and bending about the weak axis. To construct the column curve, first the tangent modulus E_t is determined for various stress levels using a stress-strain diagram such as that shown in Fig. 19a. Then a stress -vs- E_t curve would be drawn as shown in Fig. 19b. Applying the tangent modulus formula, Eq. 1, the resulting column curve of stress -vs- slenderness ratio would be

obtained, Fig. 19c. The influence of residual stresses when buckling occurs in the inelastic range is thus seen by comparing the solid line with the dashed line obtained for a member without residual stresses.

Equation 1 is valid only for the special case of a rectangle bent about the weak axis. Proceeding now to a discussion of the problem of determining the strength of an actual column, the basic equation for the critical strength of a column containing residual stresses was derived in Ref. 8 and is given by

$$P_{cr} = \frac{\pi^2 EI_e}{L^2} \quad \dots (4)$$

where I_e is the moment of inertia of that portion of the cross section which remains elastic. (See Fig. 4d, for example). In terms of the average critical stress, Eq. 4 may be written,

$$\sigma_{cr} = \frac{\pi^2 E I_e/I}{(L/r)^2} \quad \dots (5)$$

Eq. 5 is the basic equation for a column containing axially }
 symmetric cooling residual stresses. Since the flanges con-
 tribute significantly to the flexural resistance, EI , it is

evident that residual stresses in the flanges are of more pronounced influence on column strength than are residual stresses in the web.

2. Effect of Flexure Axis

There is a pronounced difference in the behavior of as-delivered columns that is dependent upon the axis about which the member bends. Columns of a given slenderness ratio in the weak direction, allowed to bend in that direction, will carry less load than columns of the same slenderness ratio in the strong direction, allowed to bend in that direction.

This difference in behavior may be shown as follows. For a rectangular section bent about the weak axis the quantity EI_e in Eq. 5 may be obtained from⁽⁸⁾

$$EI_e = E_t I \quad \dots (6)$$

and Eq. 5 would reduce to Eq. 1. Equation 6 is also very nearly true for a WF section bent about its strong axis, since the web contributes only a small portion to the moment of inertia and thus the action is similar to that of two rectangles (Fig. 4d). However, for the rectangular section bent about its strong axis and for an H-section bent about its weak axis, the term EI_e will be considerably less than

$E_t I$. Thus the buckling strength will be less than the value predicted by Eq. 1, and would be computed according to Eq. 5.

Fig. 20 illustrates this difference for an idealized case (parabolic residual stress pattern, residual stress at flange edges equal to -20 ksi, residual stress at flange centers equal to +10 ksi, yield point stress equal to 40 ksi, effect of web neglected.) The lower curve is for flexure about the weak axis of an H-section, while the upper curve is for flexure about the strong axis.

3. Effect of Stress-Strain Relationship

Columns of a material without definite yield level, and with a continuously curving stress-strain diagram show an influence of residual stress for the whole range of L/r . Although such materials can have much higher buckling strengths for a shorter column lengths (i.e. L/r 40), for the medium to long columns the effect of different stress-strain relationships is quite small, particularly in comparison with other effects such as variations in yield level and in residual stress. (10,15) Figure 21 shows column curves calculated for logarithmic, parabolic and idealized stress-strain relationships for individual fibers.

The column cross section is rectangular with a parabolic residual stress distribution.

4. Cold-Bending Residual Stresses

The study of the effect of cold-bending residual stresses on axial column strength indicates that for short and medium length columns, these stresses are no more critical than are cooling stresses.^(3,10) This means that findings based on members with cooling residual stress patterns will be conservative when applied to straight members whose cooling patterns have been modified by cold bending. (Fig. 22)

5. The Column Curve

For wide-flange shapes with axially symmetric cooling residual stresses the solution to Eq. 5 (the column curve) may be obtained either from residual stress data, or from the average stress-strain curve obtained in a stub column test.^(2,11) Both methods are essentially the same, the solution to Eq. 5 requiring the function relating σ_{cr} and the geometry of the non-yielded portion of the cross section at that particular stress, which makes possible a solution for L/r . The solution will obviously depend on the distribution of residual stress. Reference 2, 7 and 11 give

analytical expressions for this function for certain residual stress distributions.

The use of stub column data to obtain the column curve is a somewhat simpler approach. If the usual situation is assumed for a loaded WF column, namely, that the web does not yield, (or that when it does the flanges have completely yielded), then the following equations will hold true: ⁽²⁾

$$E \frac{I_{ex}}{I_x} = \frac{AE_t - 2/3 A_w E}{A_f + A_w/3} \dots (7)$$

$$E \frac{I_{ey}}{I_y} = E \left[\frac{AE_t}{A_f E} - \frac{A_w}{A_f} \right]^3$$

where A_f = area of both flanges of a WF shape and A_w = area of web.

Since for most WF columns the ratio A_f/A_w is about 3, Eqs. 7 reduce to

$$E \frac{I_{ex}}{I_x} = E (1.2 \tau - 0.2) \dots (8)$$

$$E \frac{I_{ey}}{I_y} = E \left(\frac{4}{3} \tau - \frac{1}{3} \right)^3$$

where τ is given by

$$\tau = \frac{E_t}{E} \quad \dots (9)$$

The magnitude of E_t is determined by conducting a stub column test of a WF shape containing residual stresses, plotting the average stress-strain curve, and then determining the tangent at various stress levels.

Fig. 20 was drawn using residual stress data (except that the effect of the web has been neglected). Figure 23b has been drawn using the stub column stress-strain curve of Fig. 18. In Refs. 2 and 7 the two methods were compared and were found to be in very good agreement for A7 steel. Further, the results of tests also correlate well with them.

6. Column Curve Approximations

A simplification to Eqs. 7 or 8 might be desirable for use as a basis for arriving at design formulas. It will be noted from Fig. 23b that the curve for buckling in the strong direction is approximately parabolic in shape and that for buckling in the weak direction the curve may be approximated by a straight line. Thus the two solid curves of Fig. 23b

could be replaced by the following relationships, the first of which is the same form as suggested by Bleich.⁽⁹⁾

$$\begin{aligned}\sigma_{xx} &= \sigma_y - \frac{\sigma_p}{\pi^2 E} \cdot (\sigma_y - \sigma_p) \cdot (L/r)^2 \quad \left(\frac{L}{r} \leq \pi \sqrt{\frac{E}{\sigma_p}} \right) \\ \sigma_{yy} &= \sigma_y - \frac{(\sigma_y - \sigma_p)}{\pi^2} \cdot \sqrt{\frac{\sigma_p}{E}} \cdot \left(\frac{L}{r} \right) \quad \left(\frac{L}{r} \leq \pi \sqrt{\frac{E}{\sigma_p}} \right) \dots (10) \\ \sigma_{xx} &= \sigma_{yy} = \frac{\pi^2 E}{(L/r)^2} \quad \left(\frac{L}{r} > \pi \sqrt{\frac{E}{\sigma_p}} \right)\end{aligned}$$

where σ_y is the yield stress level and σ_p is the proportional limit. The latter value could be determined either from stub column tests or from residual stress measurements using Eq. 2. A parabolic column curve, tangential to the Euler curve, was originally proposed by J. B. Johnson⁽¹³⁾ in 1893, on the basis of test results.

As an alternate design procedure, one could set up a table of $\frac{I_e}{I}$ values for use in Eq. 5. The variation of I_e/I with σ/σ_y and flexure axis is shown in Fig. 23 a and is the information used to arrive at the solid curves of Fig. 23 b.

Fig. 24 shows column curves for a number of specimens as determined by the "stub column" method and also the straight

line approximation (Eq. 10) using average values obtained for this group.* The approximation of Eq. 10 is also shown in which all of the data in Table 1 has been included. While the scatter is considerable, the straight-line approximation is evidently a good one.

Fig. 25 shows the results of weak-axis column tests in comparison with the same straight-line approximation arrived at from data from all the stub columns tested. The circles show the maximum load the columns carried, not the point of first bending. The results of annealed columns are also shown; their strength is clearly above that of the as-delivered members.

Fig. 26 shows the results of column tests and the parabolic approximation of Eq. 10 using, again, the average properties from all the stub columns tested. Again there is good correlation between theory and test.

It is evident, therefore that Eqs. 10, based on residual stress and tangent modulus considerations, could form the basis for design curves. These equations have been replotted

*In this figure, as in many that follow, the information is presented in non-dimensional form in order to adjust for variations in σ_y and E .

in Fig. 27 as Eqs. 11, 12 and 13, together with column test results. All information is presented on a dimensional basis using

$$\sigma_y = 33 \text{ ksi}$$

$$\sigma_p = 20 \text{ ksi} *$$

$$E = 30 \times 10^3 \text{ ksi}$$

in other words, the actual experimental data has been adjusted proportionately to these figures, which represent the values most appropriate as a result of this investigation. (Actually, the yield stress level, experimentally, was 33.4 ksi, but this is so close to the specification minimum of 33.0 ksi that the latter was used as the average value - particularly as the factor of safety would account for any deviation from this average.)

The test points for Fig. 27 have been plotted from the data contained in Figs. 25 and 26. For the above average values, Eq. 10 takes the form

*Using the approximate values $\sigma_p = 20$ and $\sigma_y = 33$ gives a ratio $\sigma_p/\sigma_y = 0.606$. Frequently a value of $\sigma_p/\sigma_y = 0.70$ has been used for these curves. This corresponds to $\sigma_{rc} = 0.30 \sigma_y$ which is in peak (mode) of the frequency distribution, Fig. 2. (Also, Ref. 12).

$$\sigma_{xx} = 33.0 - 8.80 \times 10^{-4} \cdot (L/r)^2 \quad \left(\frac{L}{r} \leq 122 \right) \dots (11)$$

$$\sigma_{yy} = 33.0 - 0.107 \cdot (L/r) \quad \left(\frac{L}{r} \leq 122 \right) \dots (12)$$

$$\sigma_{xx} = \sigma_{yy} = \frac{2.96 \times 10^5}{(L/r)^2} \quad \left(\frac{L}{r} > 122 \right) \dots (13)$$

In order to simplify the design procedure by eliminating the consideration of flexure axis, B. G. Johnston⁽¹⁴⁾ has suggested that $\frac{\sigma_p}{\sigma_y} = 0.5$ be used with a parabolic curve, using the single curve for both strong and weak axis bending. The resulting "CRC curve" is shown in Fig. 27 as Eq. 14,

$$\sigma_{cr} = 33.0 - 9.20 \times 10^{-4} \cdot (L/r)^2 \dots (14)$$

in general terms

$$\sigma_{cr} = \sigma_y - \frac{\sigma_y^2}{4\pi^2 E} \cdot (KL/r)^2 \dots (15)$$

In the past it was common to explain the reduction in column strength in a region up to $L/r = 100$ as due to accidental eccentricities and initial curvature. Such accidental eccentricities, in fact, were estimated and appear in certain design formulas. Although the secant type formula is derived

on the basis of an idealized stress-strain curve with yield point at the elastic limit (33,000 psi), it assumes a certain value for accidental eccentricity. This latter value was arrived at analytically by correlation with a study of column tests. Since any correlating column tests must have included as-delivered specimens that contained residual stresses, the magnitude of the accidental eccentricity or initial curvature must necessarily have been arbitrary since a considerable portion of the reduction in column strength is now known to be due to the presence of residual stresses rather than eccentricities.

Thus, design curves for column strength based on the tangent modulus method modified by the presence of residual stress, reflect actual conditions rather than a reliance on assumed irregularities.

5. BUILT-UP MEMBERS

A pilot investigation into the influence of residual stresses on the behavior of built-up columns has been carried out.⁽⁴⁾ Certain of the findings are summarized in the paragraphs that follow.

1. Residual Stresses

Welded columns will have high residual stresses; this is particularly true of the tensile residual stresses. For H-shaped members the compressive residual stresses may also be high. Figure 28 shows residual stresses in a welded H-shaped member. The tensile stresses approach the yield value and the compressive stress at the flange tips averages about 21 ksi. These stresses are compared with those in rolled shapes and in Universal plates prior to welding, in Fig. 29. It shows that compressive stresses in welded H-shaped members may be higher than those in rolled WF shapes. Compressive residual stresses in universal plates of 5 to 10 ksi have been observed in flange tips (Fig. 29).⁽⁴⁾

Since the magnitude and distribution of welded residual stresses is markedly influenced by the geometry, further work is required on members with cross-sectional shapes other than the H-section. It would be expected that the use of welded H-shape columns would be replaced more frequently in the future by the economical "box" section.

Riveted built-up columns have a considerable variation in residual stresses that is a function of the geometry

of the component parts. Fig. 30 shows some measurements that have been made.

2. Column Strength

The axial column strength of built-up H-shaped columns can be predicted with reasonable accuracy by the same techniques as were used for rolled shapes with symmetrical cooling residual stresses (see Section 4-3). The results, correlated with actual column tests, are shown in Fig. 31.

General conclusions regarding the column strength of built-up members (particularly the welded ones) cannot be made until further studies are completed. Even though the strength of the welded H-columns was proportionately less than the riveted or as-rolled columns, it is very important not to draw the conclusion that welded columns usually will be "weaker" than corresponding rolled shapes. The effect would be quite different if the cross section were in box form. Studies of the effect of cross sectional form are necessary and are underway.

6. LOW ALLOY HIGH STRENGTH STEELS

A program of tests on high strength low alloy steel (ASTM A242) has indicated that the column strength of such

steel can be predicted in the same manner as for A7 steel.⁽¹⁵⁾ The residual stress distribution in rolled shapes is the same as for A7 steel, the formation of residual stress being more dependent on shape than on variation of material properties.

Because of the high yield stress level, the influence of residual stress on columns of high strength steel is not as pronounced as on A7 steel as shown in Fig. 32.

7. SUMMARY

1. The strength of centrally-loaded steel columns may be expressed in terms of the tangent modulus E_t (Fig. 19). This modulus depends upon the state of residual stress in the member.
2. Residual stresses are formed in a structural member as a result of plastic deformations that occur during cooling after rolling, after welding, or during cold-straightening operations. For rolled or welded members, the part to cool last is usually in a tensile state of residual stress (Figs. 1 and 29).

3. Insofar as columns are concerned, the most important of the stresses are those at the flange tips; for rolled shapes the average compressive stress there is about 13.0 ksi (Table 2).

4. The effect of residual stress on the stress-strain relationship is to lower the proportional limit and to cause the stress-strain diagram to be non-linear beyond that point and up to the yield stress level (Figs. 5 and 18). The proportional limit for the shapes studied was about 21 ksi (62% of yield stress level).

5. Residual stresses reduce buckling strength because of early localized yield that occurs at certain portions of the cross section. This reduction is greatest when the slenderness ratio is between 70 and 90 and amounts to about 25% (Figs. 25 and 26). The critical stress in the inelastic range ($\sigma > \sigma_p$) is a function of the moment of inertia of the elastic portion of the cross section (Eq. 5 and Fig. 4d).

6. There is a difference in behavior of as-delivered columns that is dependent upon the axis about which the member bends. Columns of a given slenderness ratio bent about the weak axis will carry less load than columns of

the same L/r bent about the strong axis (Fig. 20). This is because the highest compressive residual stresses are found at the tips of the flanges.

7. Factors such as upper yield point, strain rate, and difference in strength between web and flange cause the yield stress level of a full cross section (stub column) to be about 20% less than the yield point determined in a routine ASTM acceptance test (Fig. 11). From a "mill test" average of all data available to date of about 42 ksi, there is obtained a probable compressive strength of about 34 ksi for A7 steel.

8. Approximate column formulas which may be adapted for design use have been developed which agree well with test results (Fig. 27). If it is desired to take into account the effect of flexure axis, then two formulas would be necessary (Eqs. 11, 12). Otherwise a single curve could be used (Eq. 15) which provides a good approximation to the theoretical and experimental results.

9. Cold-bending residual stresses are no more critical than are cooling stresses. Thus, findings based on members with cooling patterns may be applied to rolled WF columns.

(Fig. 22)

10. Columns built-up by welding will contain tensile residual stresses close to the yield point (Fig. 28). Compressive stresses may be higher or lower than those that form due to cooling, depending on the geometry of the cross section. Although tests of H-shaped welded members exhibit a strength that is comparatively less than that of a corresponding WF shape, it might be expected that welded "box" columns would be somewhat stronger than a corresponding rolled member.

11. Low-alloy, high-strength steels contain residual stresses whose magnitude and distribution are similar to those in A7 steel. Since the formation of residual stress does not depend to the same extent on the magnitude of the yield stress level as it does on the geometry, the influence of residual stress on the strength of columns of higher strength steels is not as pronounced as on columns of A7 steel. (Fig. 32)

12. Column strength is affected by the presence of residual stress, and is dependent on its magnitude and distribution. Columns of riveted and high-strength steels, for example, with proportionately smaller magnitudes of compressive residual stress, have greater strengths than columns of a welded H-shape of A7 steel. (Fig. 33)

13. Residual stresses are the major factor affecting the strength of columns in the intermediate ranges ($40 \leq \frac{L}{r} \leq 120$), and design curves for column strength based on the tangent modulus concept, modified by the presence of residual stresses, reflect actual conditions.

$\dot{\epsilon}_{pl}$	strain rate in the plastic range
σ	stress
σ_{cr}	applied average maximum stress on a column
σ_p	stress at proportional limit
σ_y	yield stress level
σ_{ys}	static yield stress (zero strain rate)
σ_{rc}	residual stress at flange edges
σ_{rw}	residual stress at web center
σ_{ro}	residual stress at flange center

2. Definitions

Buckling:

Buckling is the process for any structure or part of a structure to pass from one deflection pattern into another without a change of load.

Critical Load:

The maximum load a column will carry. It is not coincident with the buckling load for an axially loaded column.

Plastic Strain Rate:

The strain rate in the plastic range. For a coupon

test, the plastic strain rate, unlike the elastic strain rate, is independent of the elasticity of the testing machine.

Stub Column:

A short compression test specimen, sufficiently long for use in measuring the stress-strain relationship for the complete cross section, but short enough to avoid buckling in the elastic and plastic ranges.

Yield Point:

The first stress in a material, less than the maximum attainable stress, at which an increase in strain occurs without an increase in stress.

Yield Stress:

The stress at which a material exhibits a specified limiting deviation from the proportionality of stress to strain.

Yield Stress Level:

The average stress during actual yielding in the plastic range. It remains fairly constant provided the strain rate remains constant.

9. ACKNOWLEDGEMENTS

This report summarizes the theoretical and experimental studies made during the course of a research program on the influence of residual stress on column strength carried out at the Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania, of which William J. Eney is director.

The Pennsylvania Department of Highways and the Bureau of Public Roads, the National Science Foundation, and the Engineering Foundation through the Column Research Council sponsored jointly the research program.

Since this report is essentially a summary of progress reports prepared on the investigation, acknowledgement is due those other investigators at Fritz Laboratory to whose work reference is made throughout the report, in particular, to Dr. Alfons Huber who conducted the earlier phases of this research program.

TABLE 1 - TEST RESULTS

Test No.	Shape	Yield Stress Level (σ_y)				Prop. Limit (σ_p) Stub Column Test	Resid. Stress (σ_{rc})	Column Test	
		Mill Test	Sim. Mill Test	Coupon Test	Stub Column Test			σ_{cr}/σ_y	L/r
T-0	8WF31	43.3		41.4	37.3	25.6	-13.9		
205A-T18	8WF31							0.99	28(x)
205A-T15	8WF31							.905	42(x)
T-1	8WF31			37.0	36.2	28.0	-11.5		
205A-T11	8WF31							.935	56(x)
T-2	8WF31	43.3		40.2	37.8	25.2	-12.5		
205A-T25	8WF31							.74	82(y)
T-3	8WF31*			36.0 7	33.4	26.0			
T-4	8WF31	43.3		40.1 7	36.7	27.0	- 9.0	.815	58(y)
T-5	8WF31*			33.9 7	31.4	31.0			
T-5 a	8WF31*							.925	82(y)
T-5 b	8WF31*							.98	58(y)
T-6 a	8WF31						-16.1		
T-6 b	8WF31						(Average)		
T-6 c	8WF31								
T-7	8WF24	39.8	40.7	36.1	35.0	26.4	-10.2	.73	84(y)
T-8 a	8WF67	43.0	37.5	31.4	31.4	25.4	- 9.5	.815	85(y)
T-8 b	8WF67						- 8.0		
T-9 a	12WF50	42.6	41.3	37.6	35.8	24.0	- 5.5		
T-9 b	12WF50							.81	80(y)
T-10	12WF65	39.7	40.7	38.3	36.6	22.0	-18.7	.76	81(y)
205E-D1	10WF33	42.9			34.4	25.8			
205E-D2	8WF24	39.8	40.4	34.2	33.4				

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TABLE 1 - TEST RESULTS (Cont'd.)

Test No.	Shape	Yield Stress Level (σ_y)				Prop. Limit (σ_p)	Resid. Stress (σ_{rc})	Column Test	
		Mill Test	Sim. Mill Test	Coupon Test	Stub Column Test			σ_{cr}/σ_y	L/r
205E-D3	10WF39	41.2	41.0	35.6	35.9				
205E-D4	12WF50	42.6	42.0	37.2	36.3				
205E-D5	8WF35	40.0	40.8	37.6	36.7				
205E-D6	10WF21	46.9	48.1	41.6	37.2				
205A-T25	4WF13			41.1	39.8		-10.0	0.78	91(x)
205A-T28	4WF13							.49	118(x)
T-11	6WF15.5	44.0	41.0	36.4	43.0	20.7		.73	87(y)
T-12 a	5WF18.5	42.2	36.6	33.8	36.7	27.5	- 7.7	.81	78(y)
T-12 b	5WF18.5							.78	80(y)
T-12 c+	5WF18.5						-10.6	.95	78(y)
T-12 d	5WF18.5								
T-13	14WF426	35.9	31.5	28.6			-17.8		
T-14	6WF15.5	49.1					-15.1		
T-15 a	14WF43	41.6					- 8.5		
T-15 b	14WF43	41.6					- 8.5		
T-16	36WF150	47.0					-10.8		
T-17	12Jr14	50.4					- 4.1		
T-19	6WF25	36.7	42.2	35.0					
T-22	10WF66	46.8	38.8	32.4	33.2	22.0		.82	72(x)
T-23	10WF66	43.9		35.5					
T-24	5WF18.5	48.8	44.7	41.3	38.7	32.3			
T-25	14WF111	45.0	39.4	32.7	33.0	22.8			
T-26 a	14WF111		43.8	33.9	35.5	16.3	-14.0	.73	60(y)
T-26 b	14WF111							.58	99(y)

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TABLE 1- TEST RESULTS (Cont'd.)

Test No.	Shape	Yield Stress Level (σ_y)				Prop. Limit (σ_p)	Resid. Stress (σ_{rc})	Column Test	
		Mill Test	Sim. Mill Test	Coupon Test	Stub Column Test			σ_{cr}/σ_y	L/r
T-27	6WF15.5	51.1	52.1	43.3	43.0	19.7			
T-28	8WF24	47.4	48.5	37.8	39.4	15.0			
T-29	8WF31	44.4	48.8	37.9	36.1	29.5			
T-30	8WF35	48.3	44.7	35.3	35.9	20.0			
T-31	8WF67	33.5	34.7	26.3	26.4	17.5			
T-32	10WF33	52.0	44.3	34.3	32.4	21.3			
T-33	10WF39	41.9	44.7	34.7	37.2				
T-34	12WF50	42.2	43.1	34.4	32.9	16.4			
T-35	12WF65	44.3	38.6	33.8	32.6	18.0			
T-36	12WF53	44.9	46.3	34.4	35.0	21.7			
T-37	12WF53	35.1	40.4	35.2	35.0	22.7			
T-38	12WF92	45.7	41.4	33.6	34.4	14.6			
T-40	12WF190	34.1	32.9	26.8	24.6	12.5			
T-41	12WF190	39.6	39.2	30.9	30.2	18.9			
T-42	14WF53	37.1	36.9	29.6					
T-44	14WF61	44.3	35.7	30.6					
T-45	14WF61	44.2	42.7	36.3	36.7	27.6			
T-46	14WF78	38.4	33.6	29.2	29.4	19.2			
T-47	14WF78	42.3	44.2	35.1	35.8	21.0			
T-48	14WF142	37.1	38.9	29.3	30.7	18.7			
T-49	14WF142	51.2	45.2	38.0	38.7	20.6			
T-50	14WF228	38.2	35.2		25.8	16.3			
T-52	14WF320	38.5	26.4	22.7					

TABLE 1 - TEST RESULTS (Concl'd.)

Test No.	Shape	Yield Stress Level (σ_y)				Prop. Limit (σ_p) Stub Column Test	Resid. Stress (σ_{rc})	Column Test	
		Mill Test	Sim. Mill Test	Coupon Test	Stub Column Test			σ_{cr}/σ_y	L/r
T-54	14WF426	38.2	34.1						
T-56	16WF88	42.3	38.3	31.4	31.4	12.8			
T-57	16WF88	41.6	39.8	34.3	34.4	11.1			
T-58	18WF105	43.1	40.6	30.4	29.8	17.0			
T-59	18WF105	37.7	38.0	32.8	33.0	19.6			
249-R1					35.3	23.7		0.86	80(x)
249-R2	Built-			36.7*	37.3	22.5		.89	60(y)
249-R3	up							.80	79(x)
249-W1					34.3	13.7		.62	74(x)
249-W2	Column				32.7	12.5	-21.0	.64	55(y)
249-W3				31.7*				.50	98(y)

NOTE:

* (Shape) "Annealed" material; others, "as delivered"

Yield Stress Level (Coupon Test) value means the weighted average value of Tension Coupon Test Results, except those marked "*", which are of Compression Test Results. "Coupon Test" were for the static yield stress

Column Test: "(x)" means buckling about strong axis
 "(y)" means buckling about weak axis

TABLE 2 - RESIDUAL STRESSES IN WF SHAPES DUE TO COOLING

	Flange Edge (σ_{rc})			Flange Center (σ_{ro})			Web Center (σ_{rw})		
	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.
Columns $d/b \leq 1.5$	-7.7	-12.8	-18.7	+16.5	+ 4.7	-4.1	+18.2	+ 8.0	-15.5
Beams $d/b > 1.5$	-4.1	- 7.5	-10.8	+24.2	+15.1	+8.3	- 8.8	-21.8	-41.0

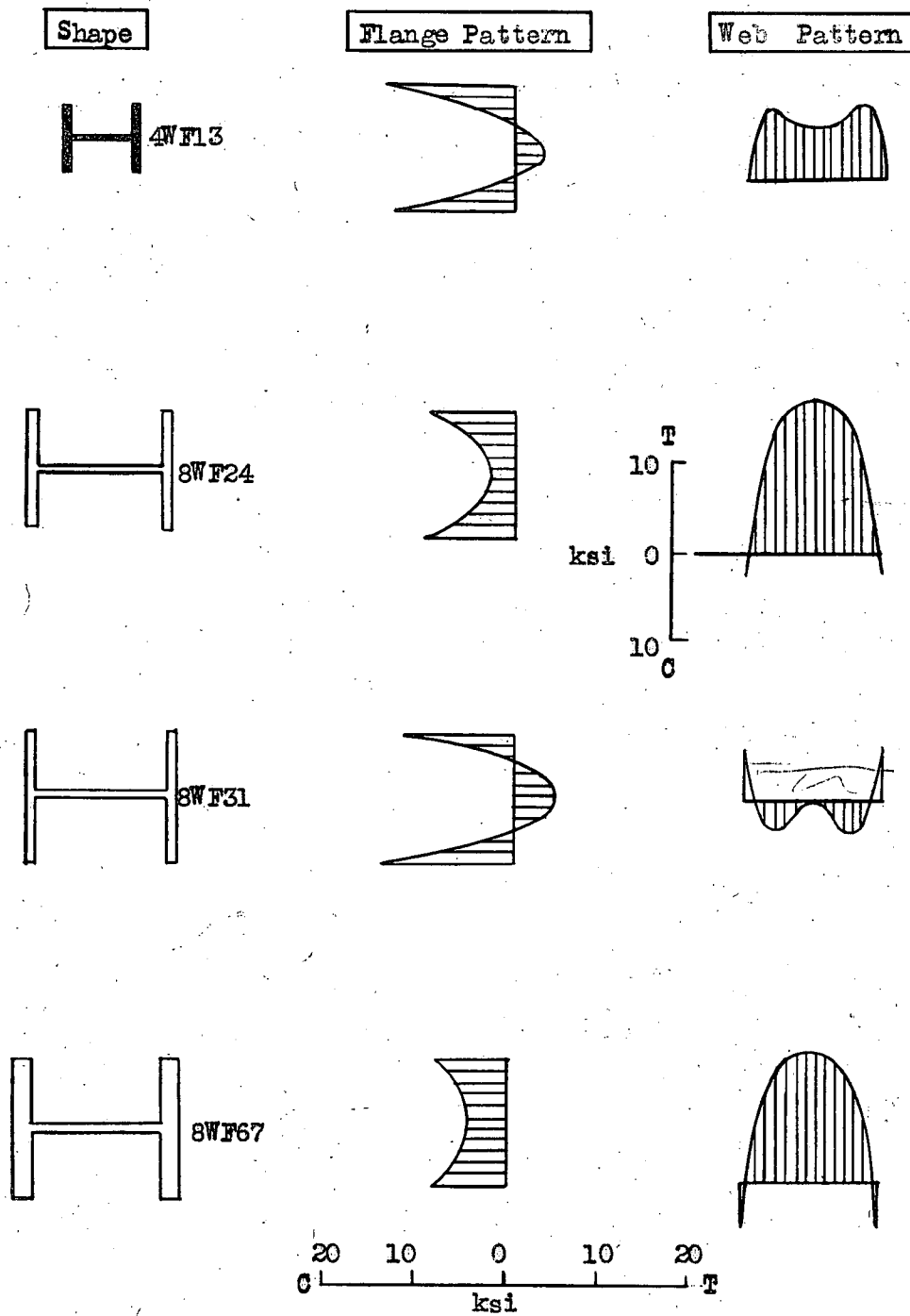


FIG. 1a RESIDUAL STRESS DISTRIBUTION IN WF SHAPES

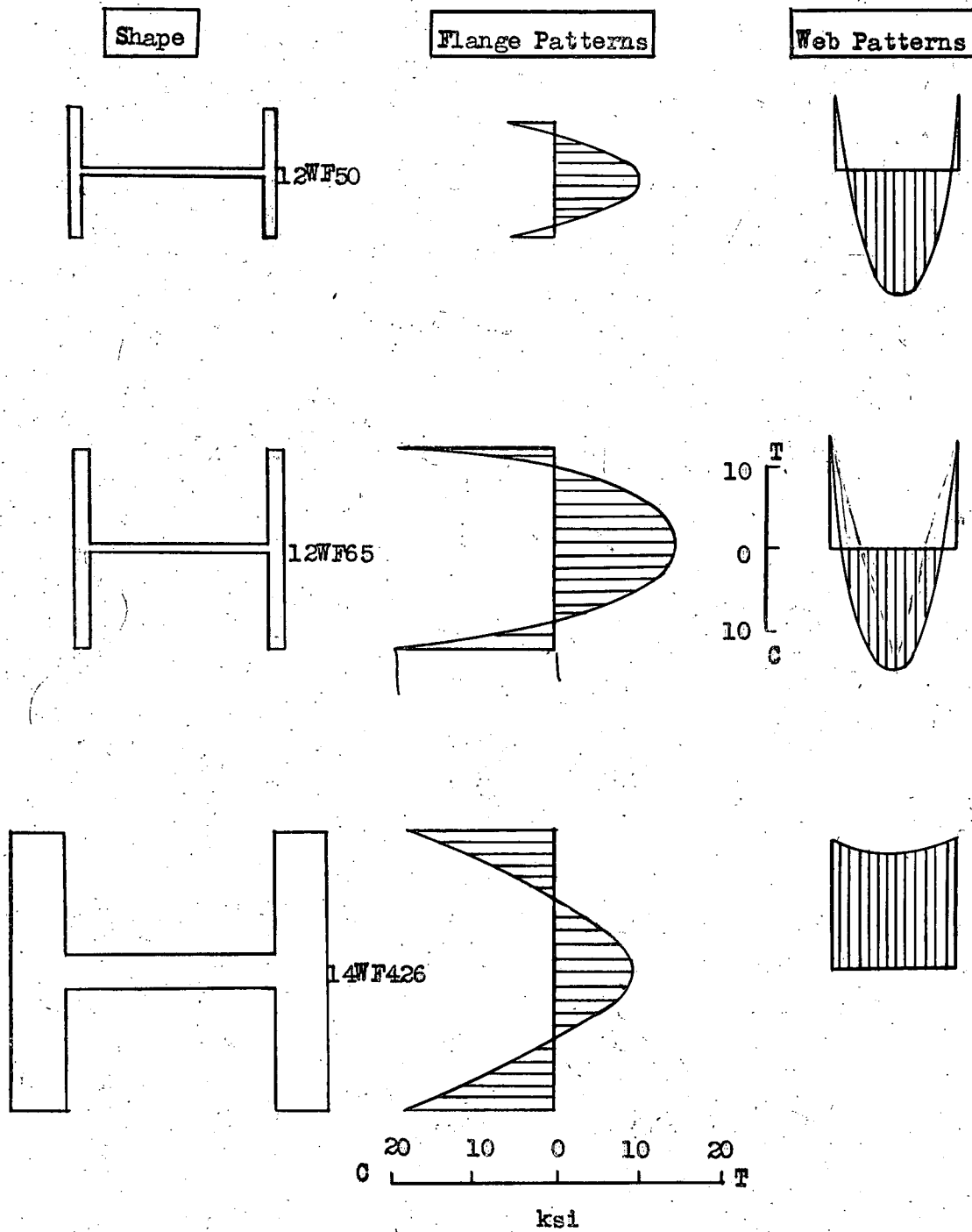


FIG. 1b RESIDUAL STRESS DISTRIBUTIONS IN WF SHAPES

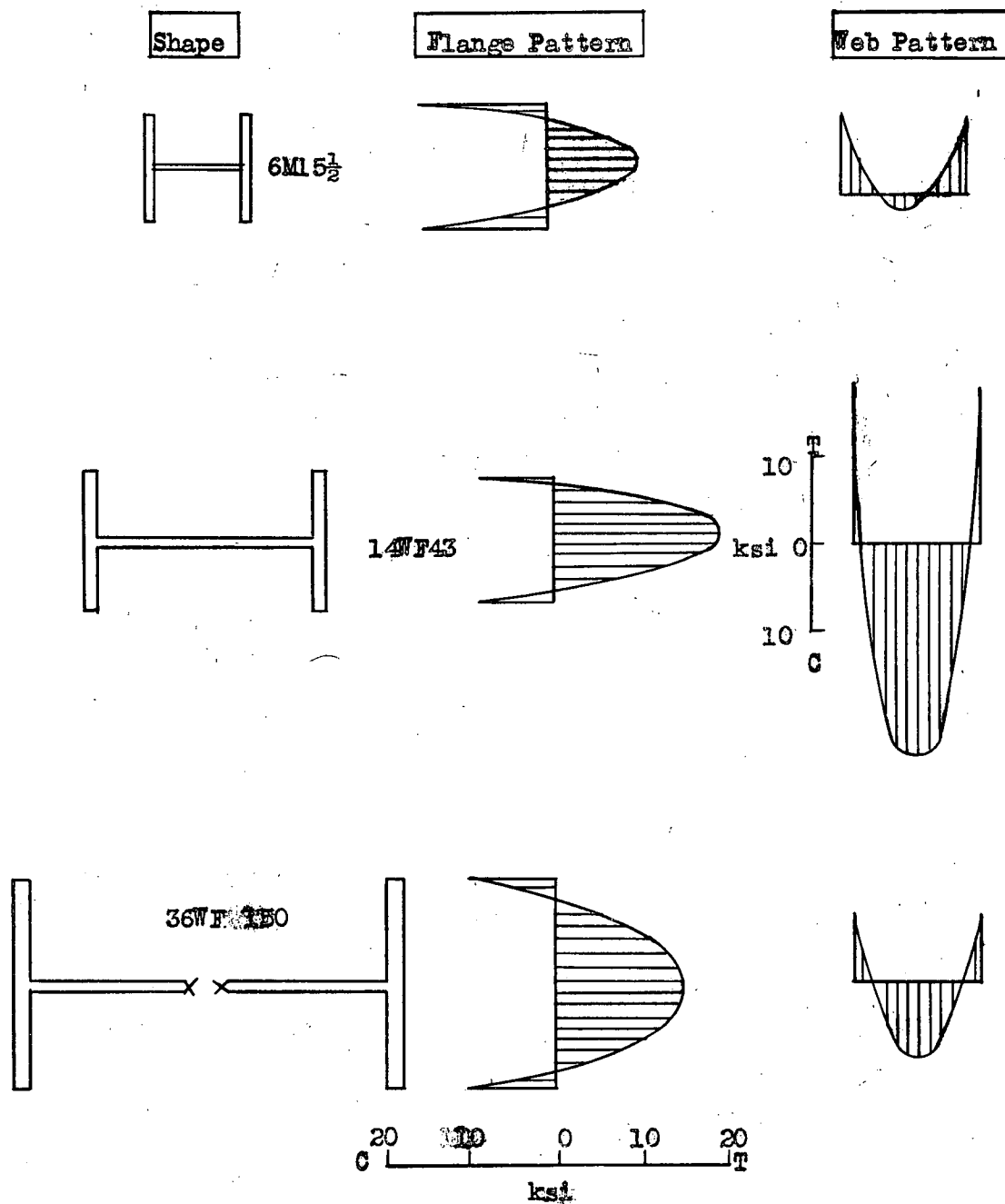


FIG. 1c RESIDUAL STRESS DISTRIBUTIONS IN WF SHAPES

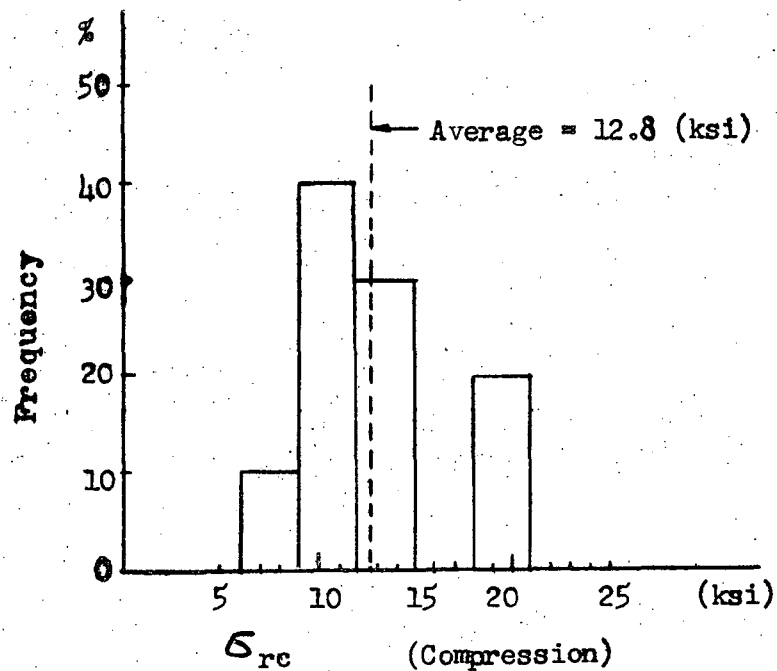


FIG. 2 (a) RESIDUAL STRESS AT FLANGE TIPS MEASURED IN SPECIMENS

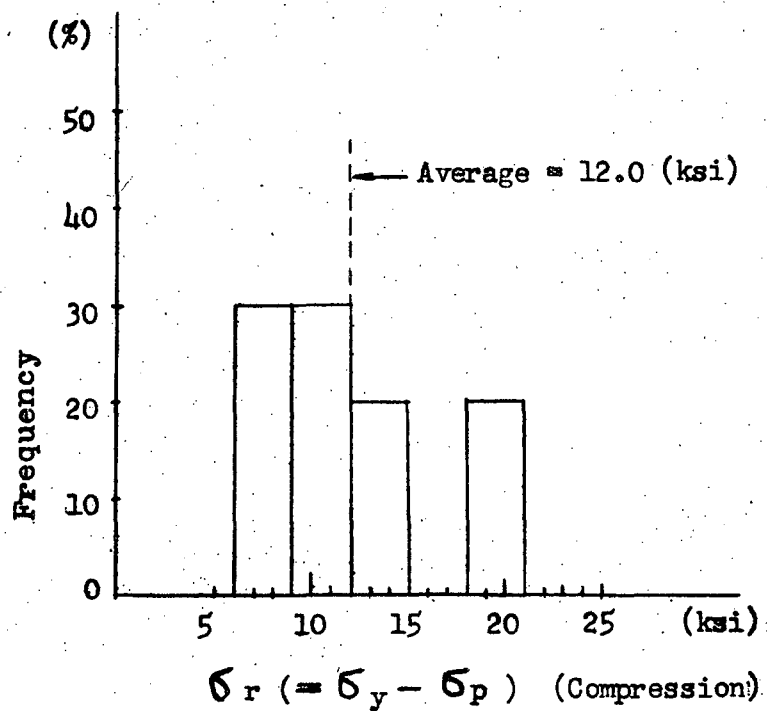


FIG. 2 (b) RESIDUAL STRESS MEASURED BY STUB COLUMN TESTS

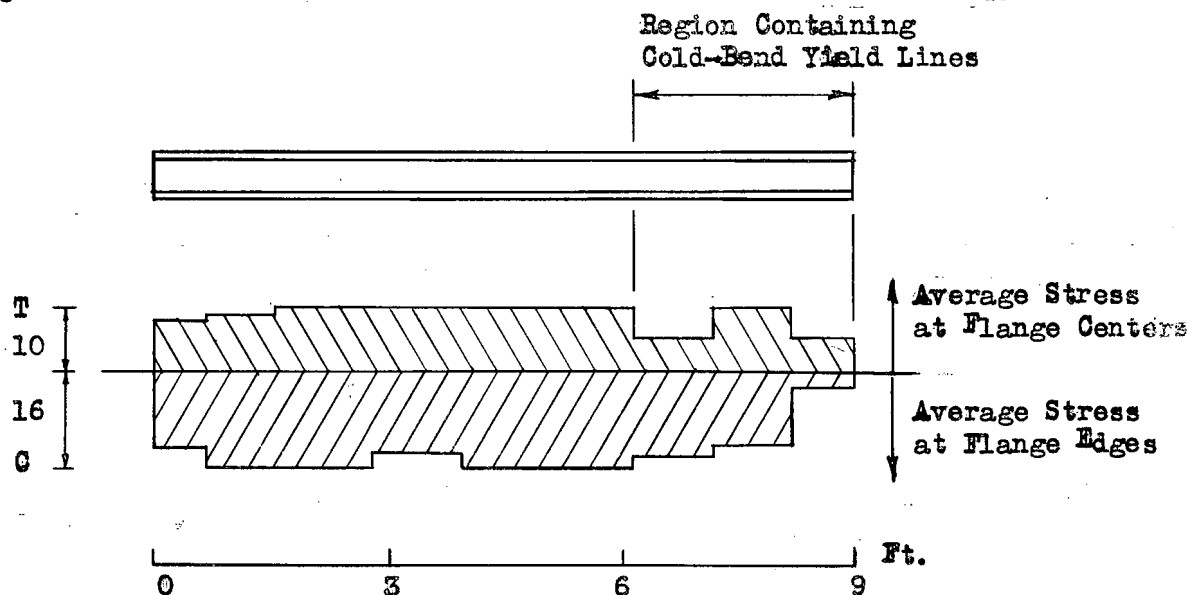


FIG. 3(a) VARIATION OF RESIDUAL STRESS (KSI)
ALONG WFF31 BEAM

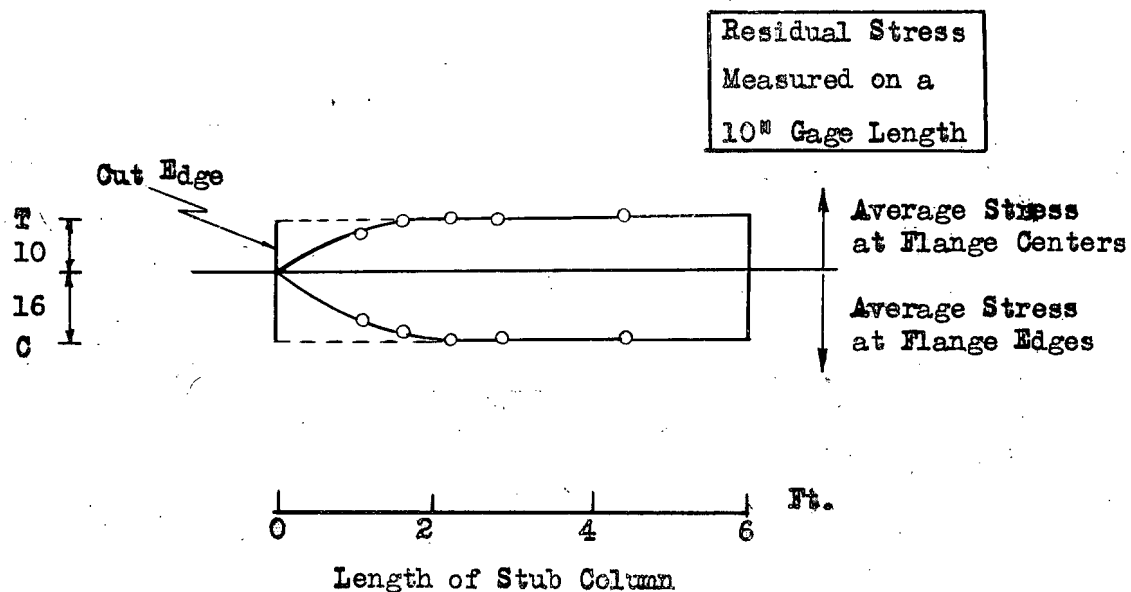
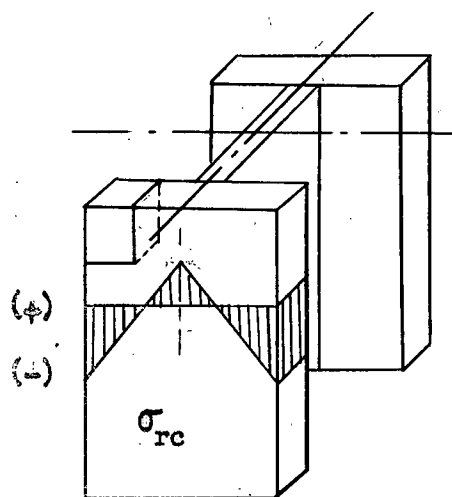
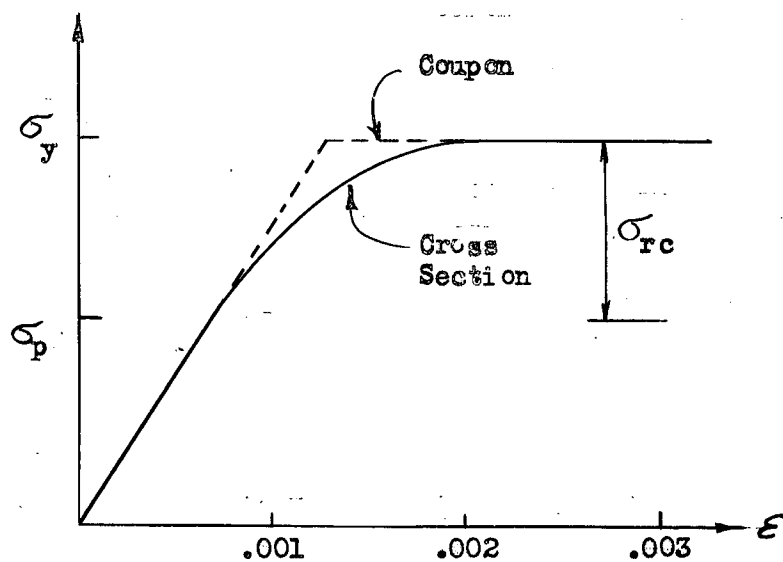


FIG. 3(b) VARIATION OF RESIDUAL STRESS (KSI)
WITH STUB COLUMN LENGTH
(MEASURED OVER MIDDLE 10")

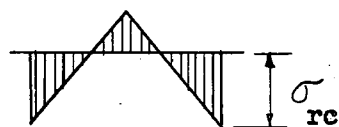


(a)

Average Stress



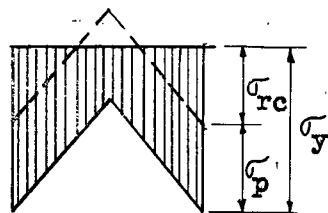
(b)



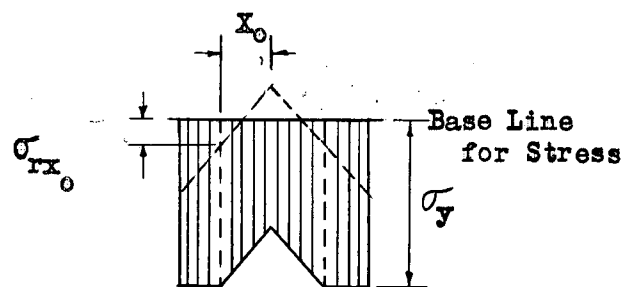
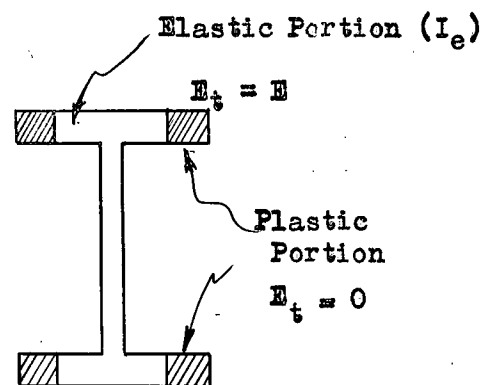
+



=



(c)



(d)

FIG. 4 INFLUENCE OF RESIDUAL STRESS ON THE STRESS-STRAIN CURVE

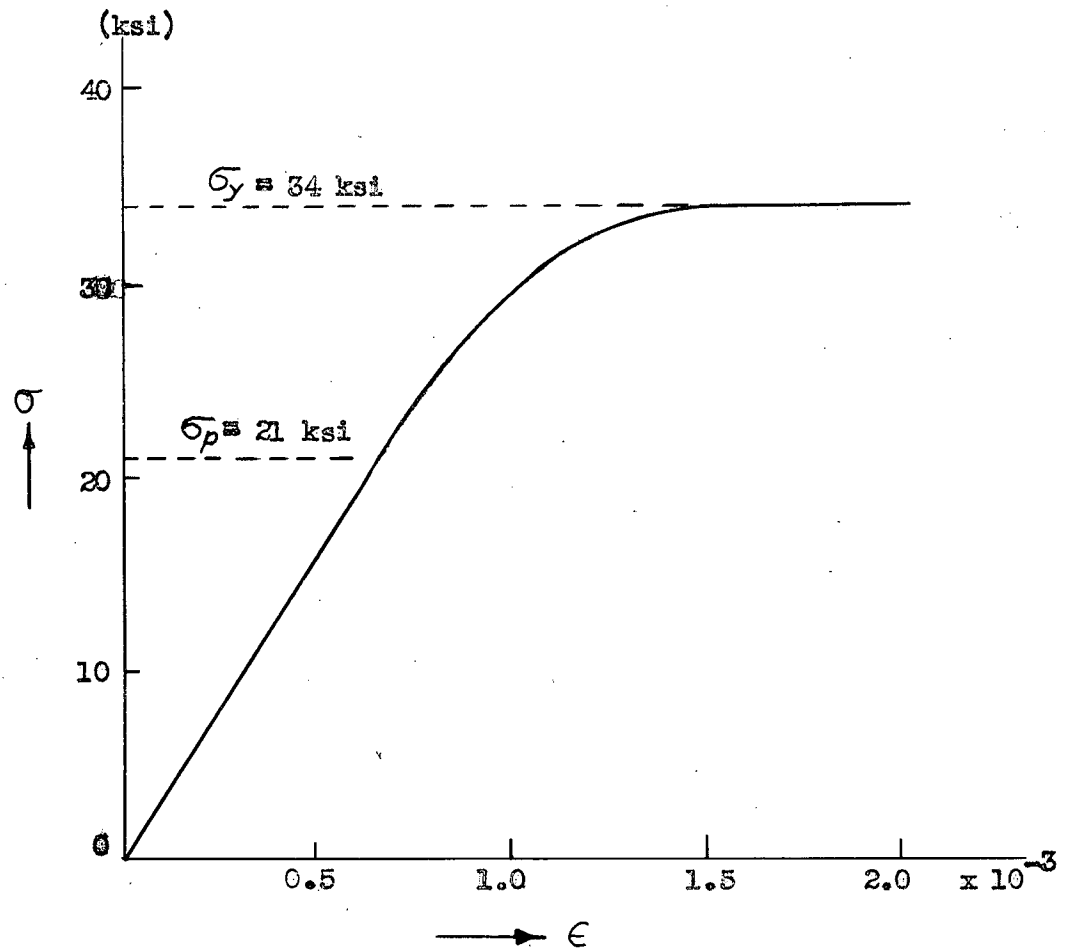


FIG. 5 STRESS-STRAIN CURVE FOR ASTM A7 COLUMNS
BASED ON MEASURED RESIDUAL STRESSES

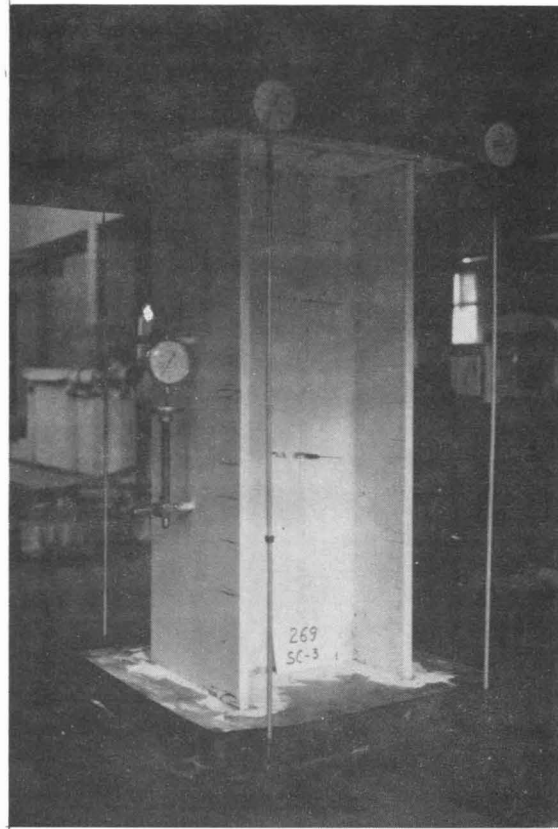


FIG. 6 TYPICAL STUB COLUMN TEST

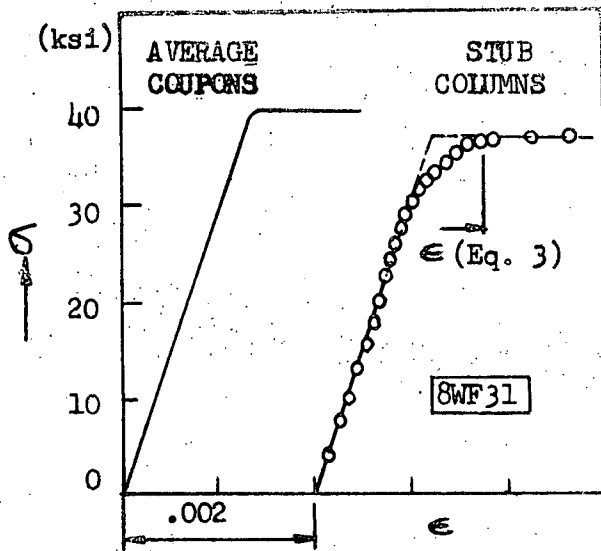


FIG. 7 STUB COLUMN STRESS-STRAIN CURVE FOR AS-DELIVERED MATERIAL

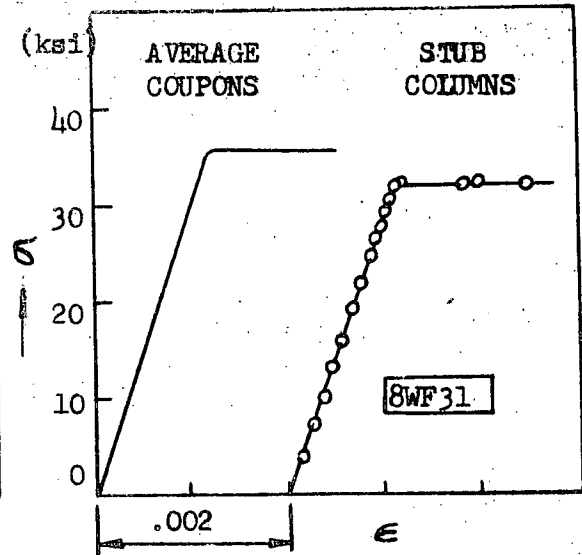


FIG. 8 STUB COLUMN STRESS-STRAIN CURVE FOR ANNEALED MATERIAL

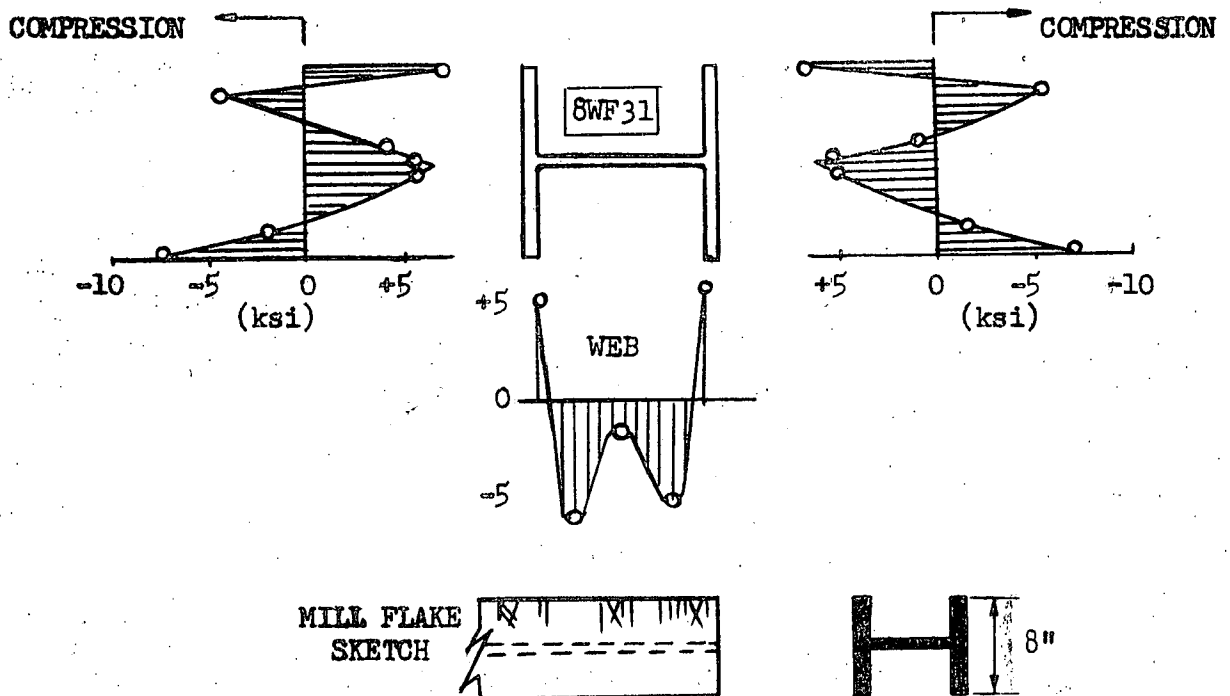


FIG. 9 COLD-BENDING RESIDUAL STRESS IN 8WF31 SHAPE

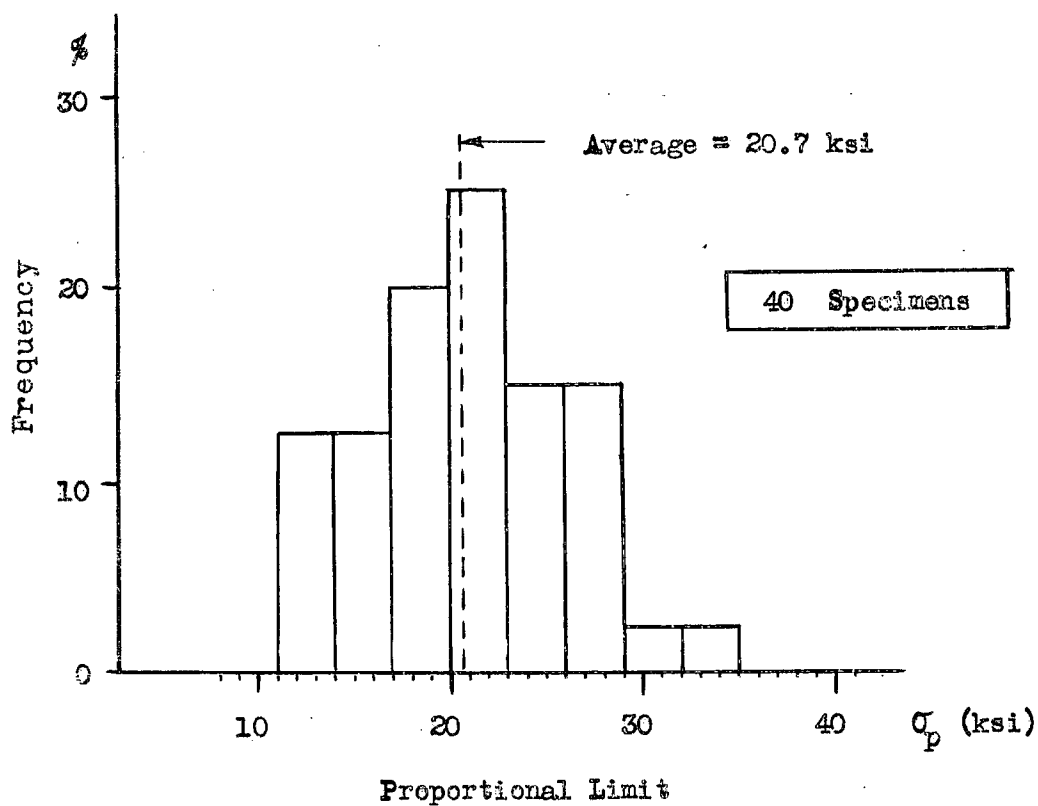
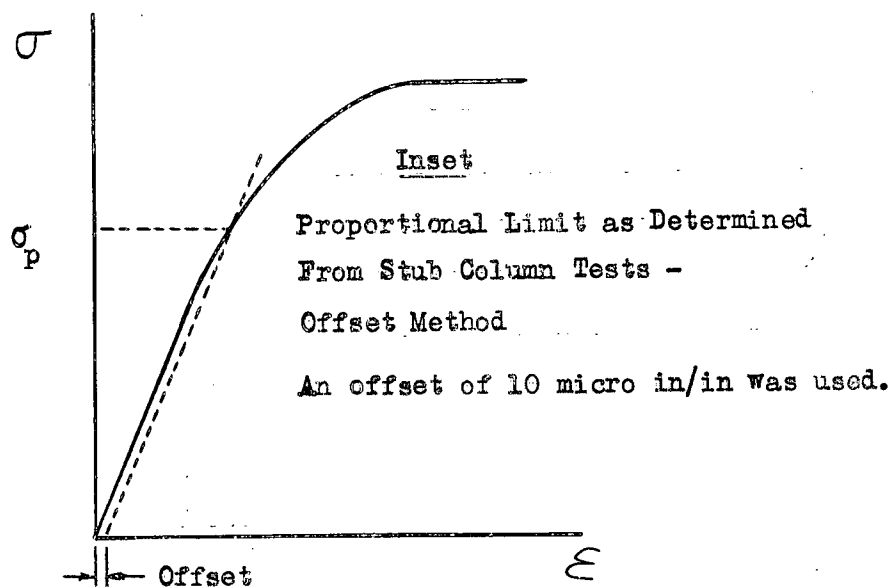


FIG. 10 FREQUENCY DISTRIBUTION OF THE PROPORTIONAL LIMIT DETERMINED FROM STUB COLUMN TESTS

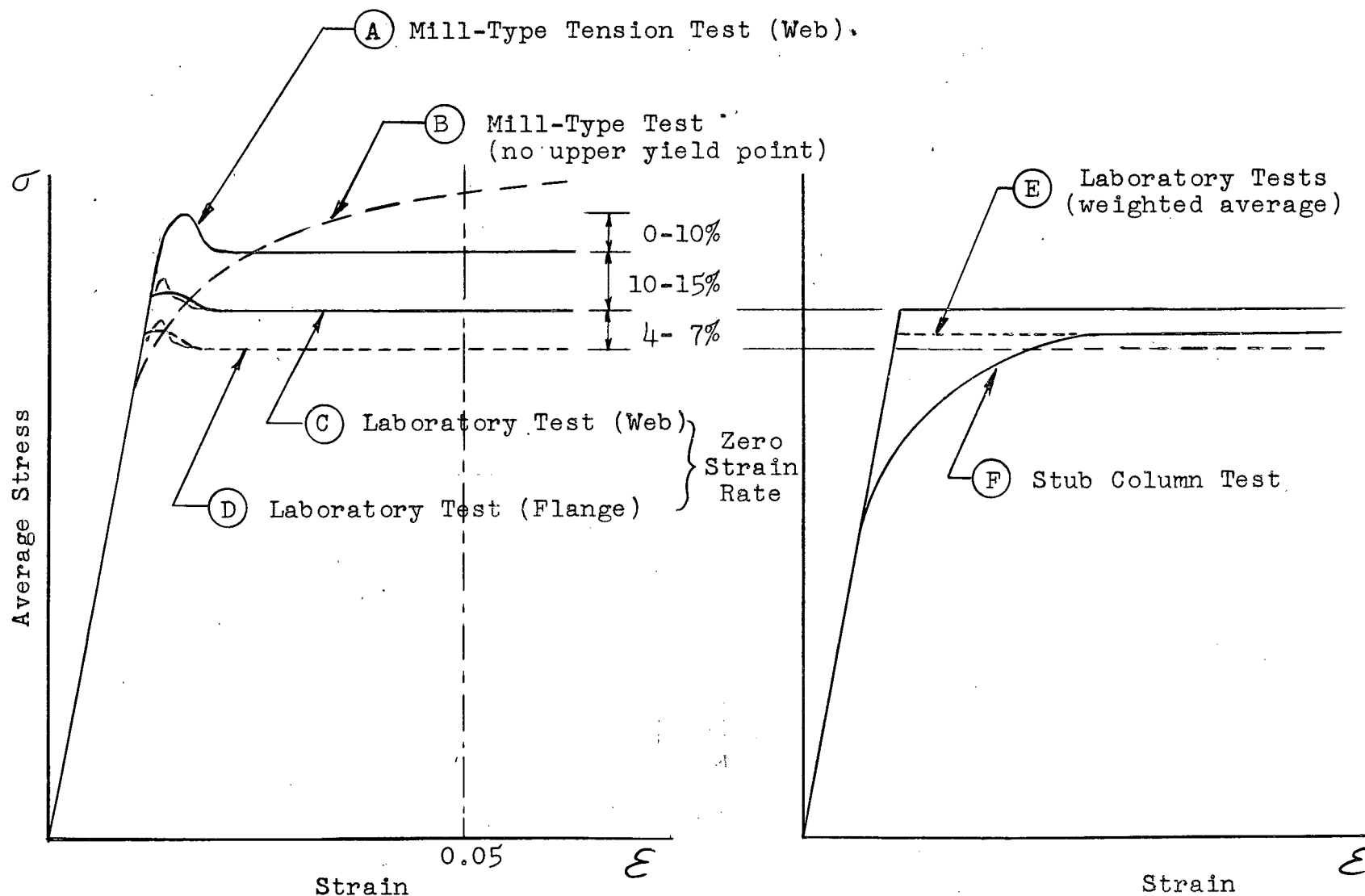


FIG. 11 INFLUENCE OF SEVERAL VARIABLES ON THE YIELD STRESS LEVEL, σ_y

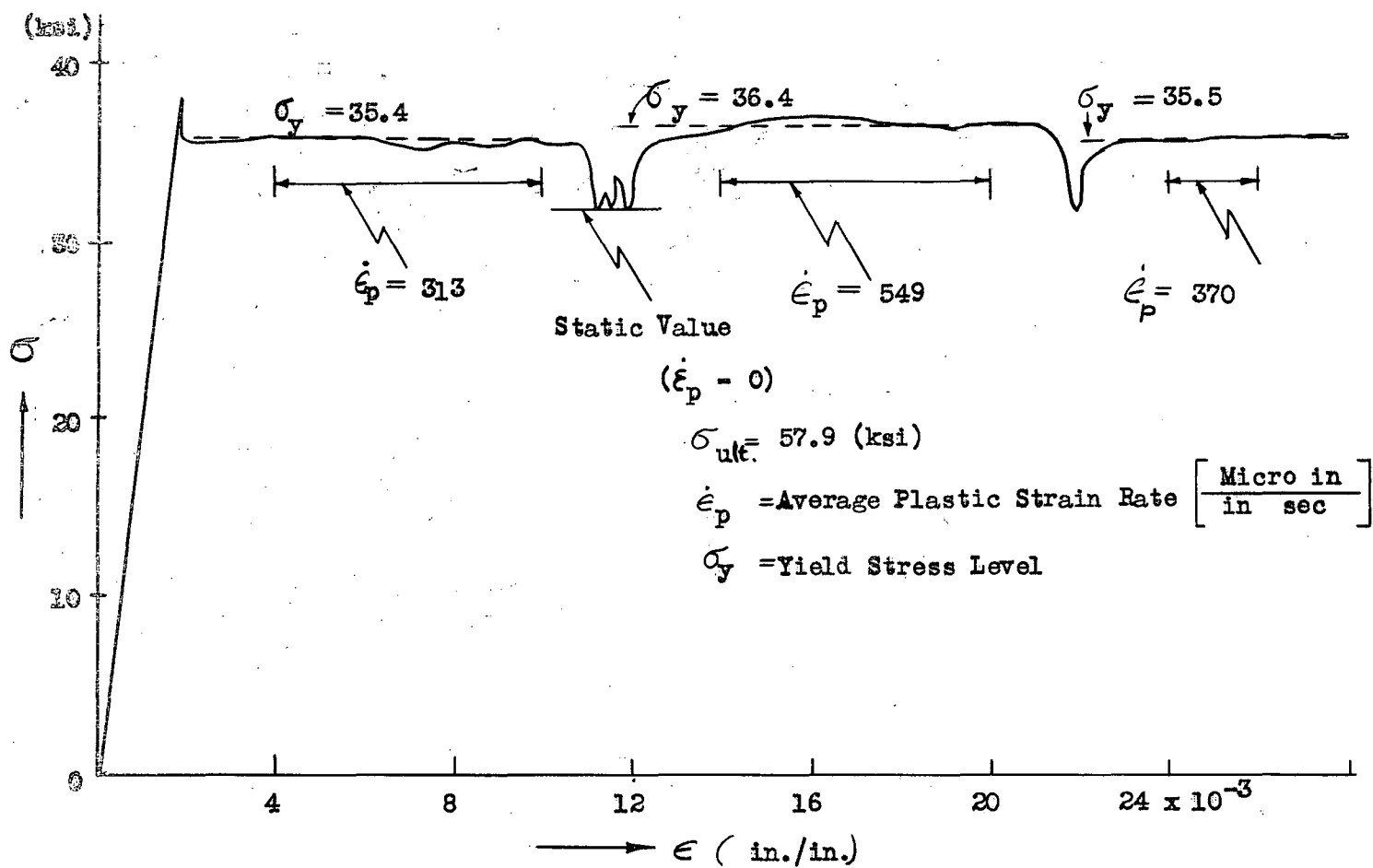


FIG. 12 STRESS-STRAIN CURVE SHOWING INFLUENCE OF STRAIN RATE

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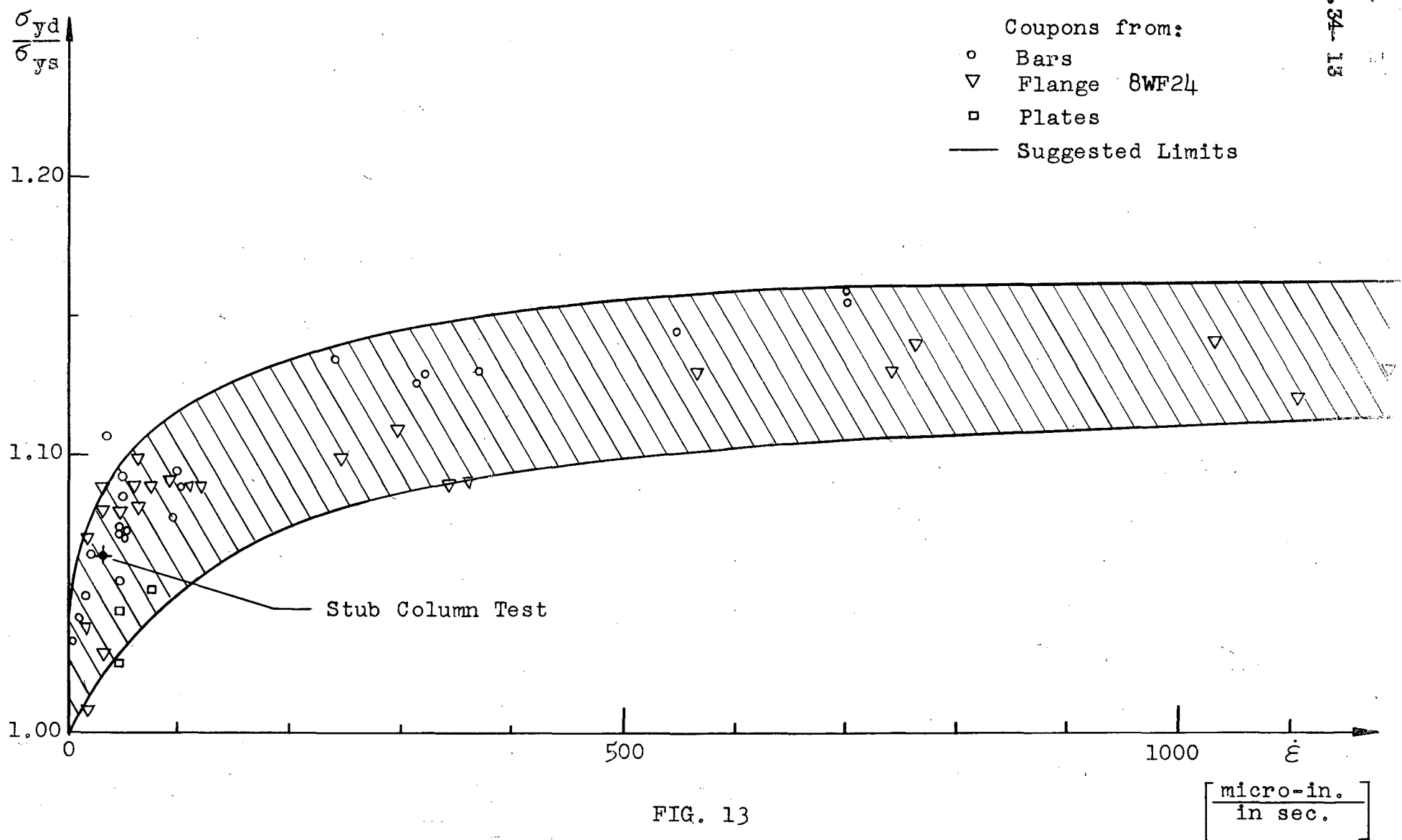
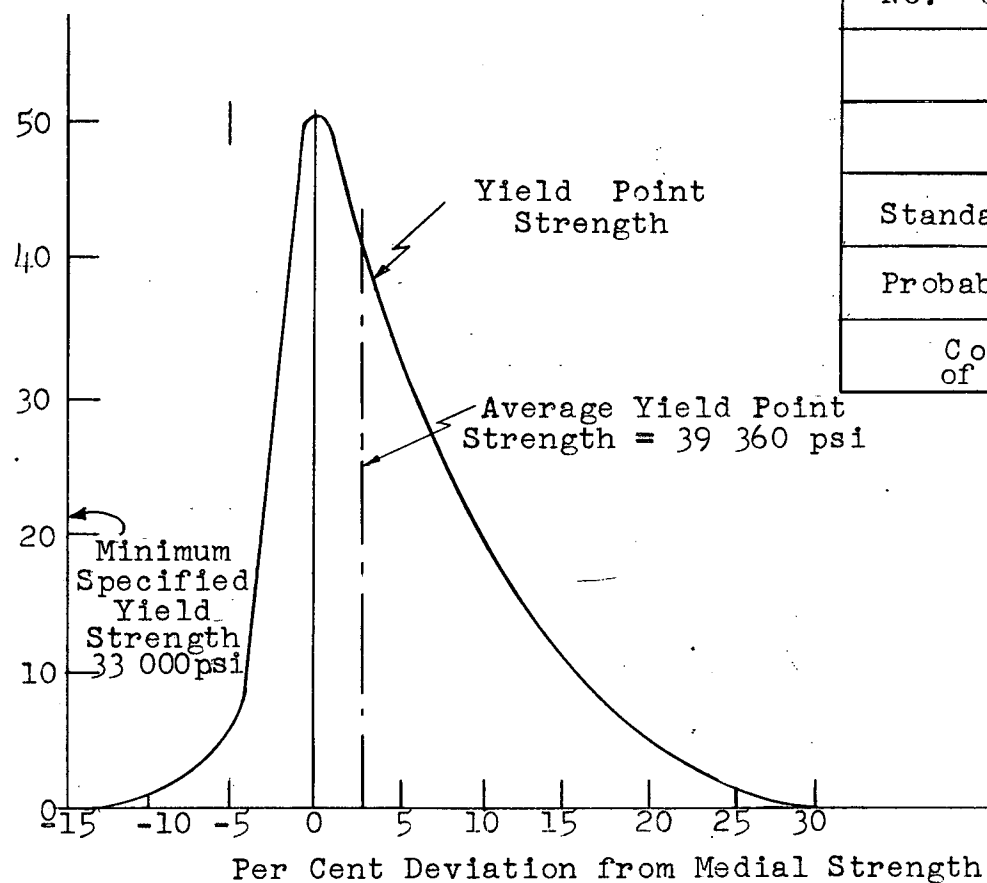


FIG. 13

$\frac{\sigma_{yd}}{\sigma_{ys}}$ As A FUNCTION OF THE STRAIN RATE

PERCENTAGE PROBABILITY OF INDICATED DEVIATION FROM MEDIAN BEING EQUALLED OR EXCEEDED NUMERICALLY (ACCORDING TO WHETHER CURVE IS TO THE LEFT OR RIGHT OF ZERO DEVIATION).



	Yield Point
No. of Mill Tests	3 124
High	56 650 psi
Low	31 090 psi
Standard Deviation	3 127 psi
Probability Error	2 109 psi 5.5%
Coefficient of Variation	7.8%

2 14 186 1360 611 347 273 204 69 37 14
Number of Tests Per 5% Bracket

FIG. 14 - CUMULATIVE FREQUENCY DISTRIBUTION OF THE YIELD STRESS (MILL TEST)

①	Total (3010 specimens)
②	$0.3'' < t < 1.0''$ (1699 specimens)
③	$t \leq 0.3''$ (1311 specimens)

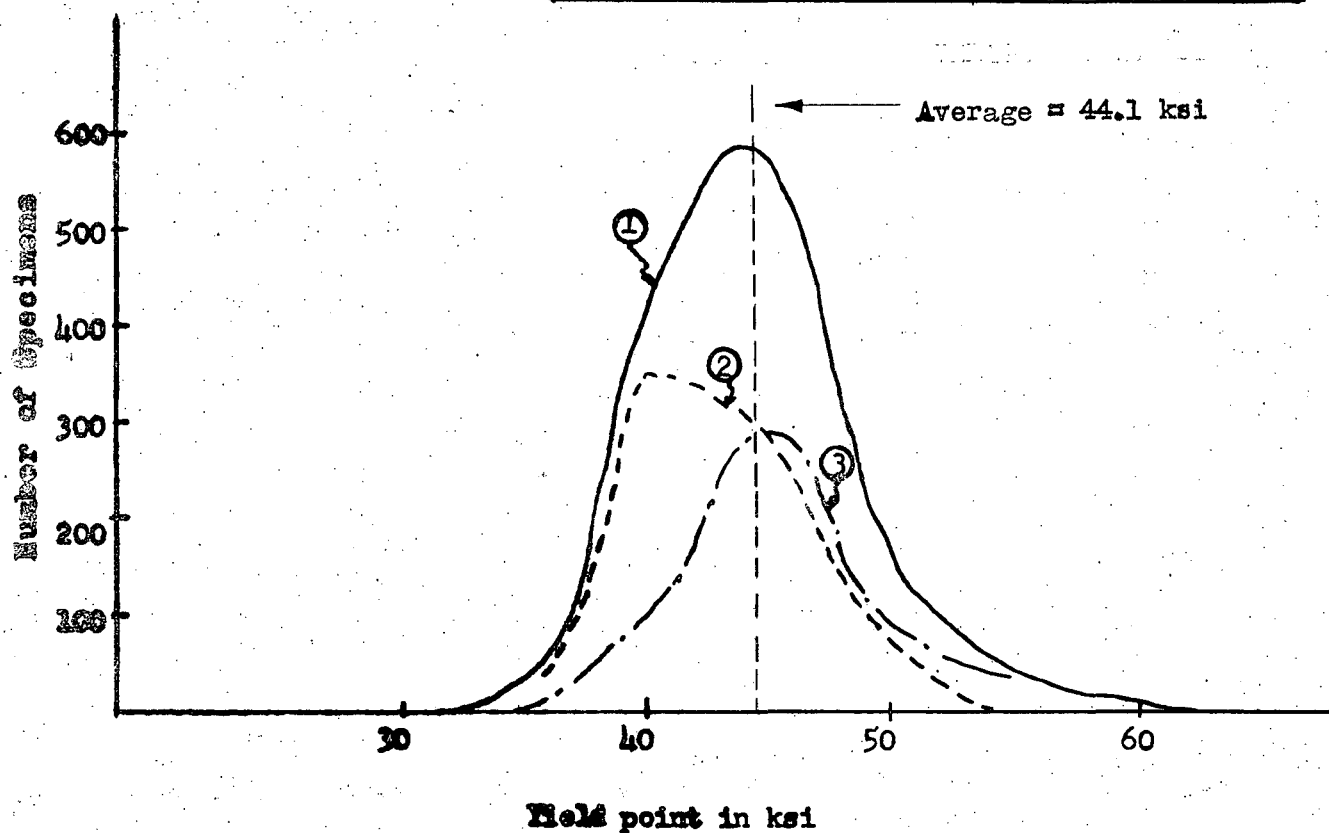
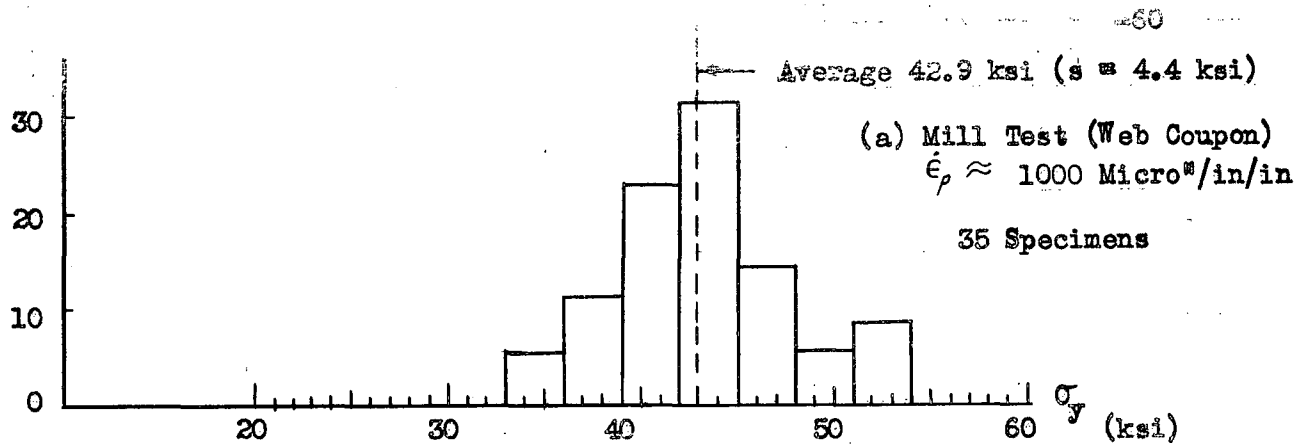


FIG. 15 MILL TEST YIELD POINT FREQUENCY DISTRIBUTION CURVES

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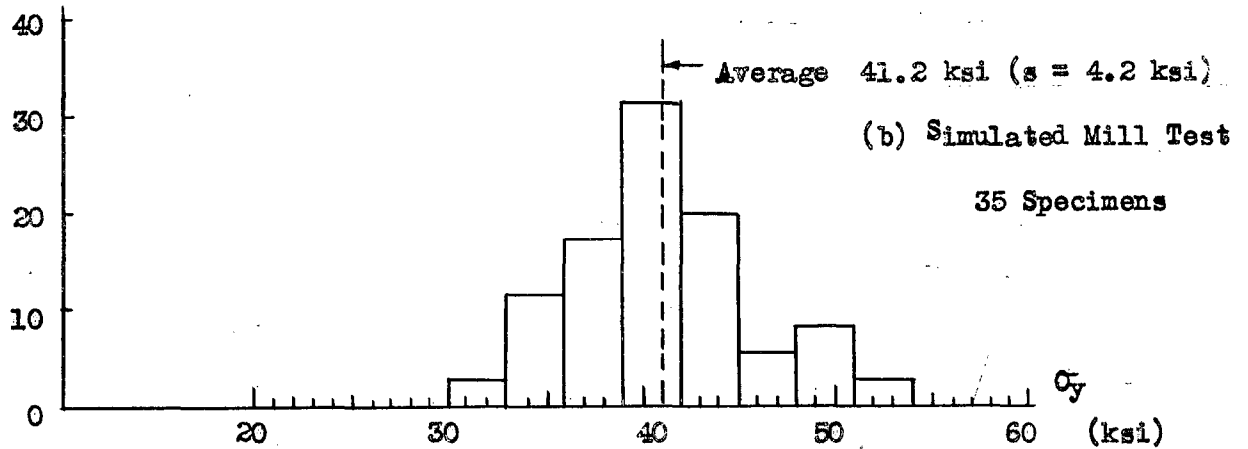
(%)

Frequency



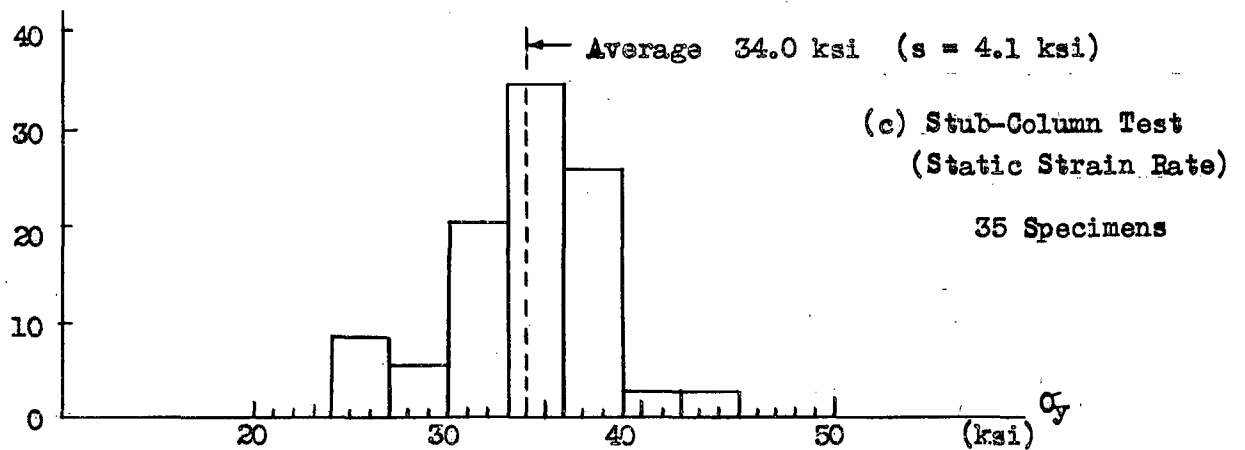
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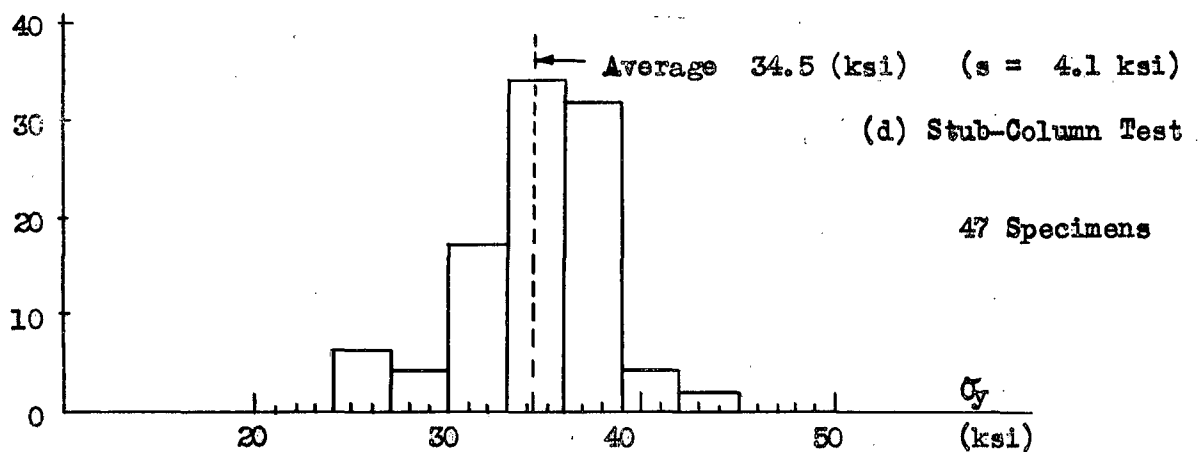


FIG. 16 YIELD STRESS LEVEL AS DETERMINED BY VARIOUS METHODS

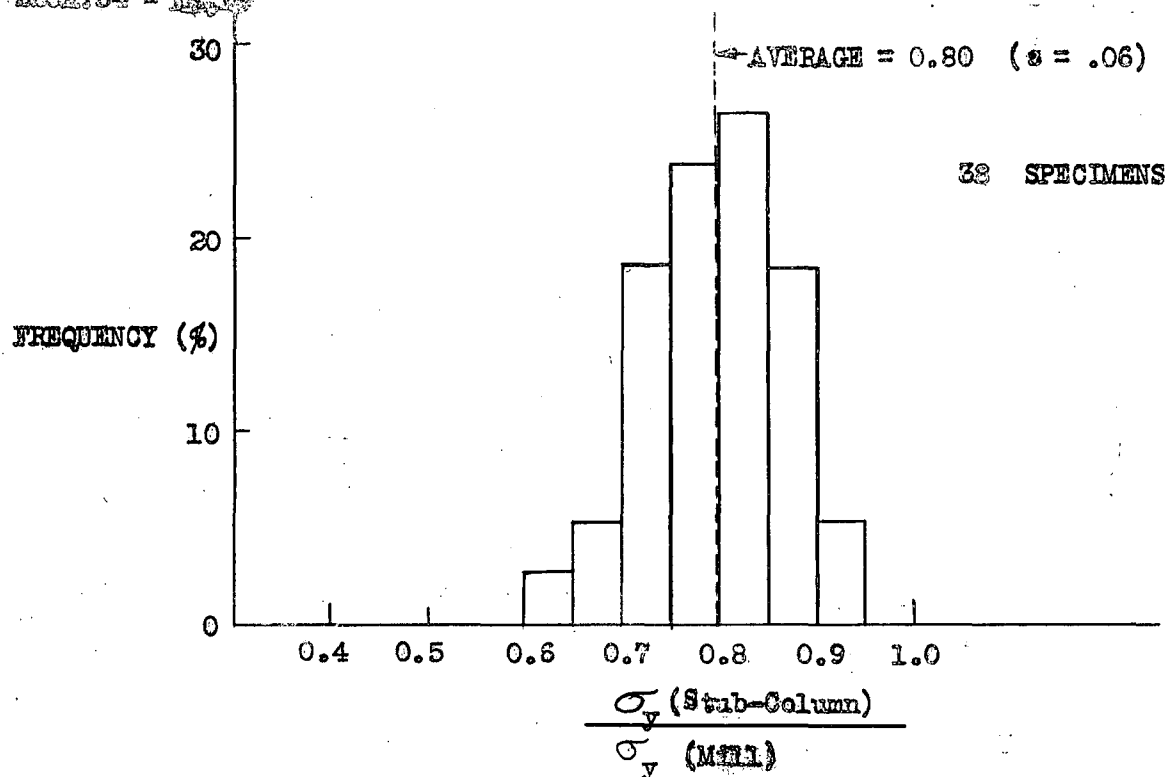


FIG. 17 FREQUENCY DISTRIBUTION OF RATIO $\frac{\sigma_y \text{ (Stub-Column)}}{\sigma_y \text{ (Mill)}}$

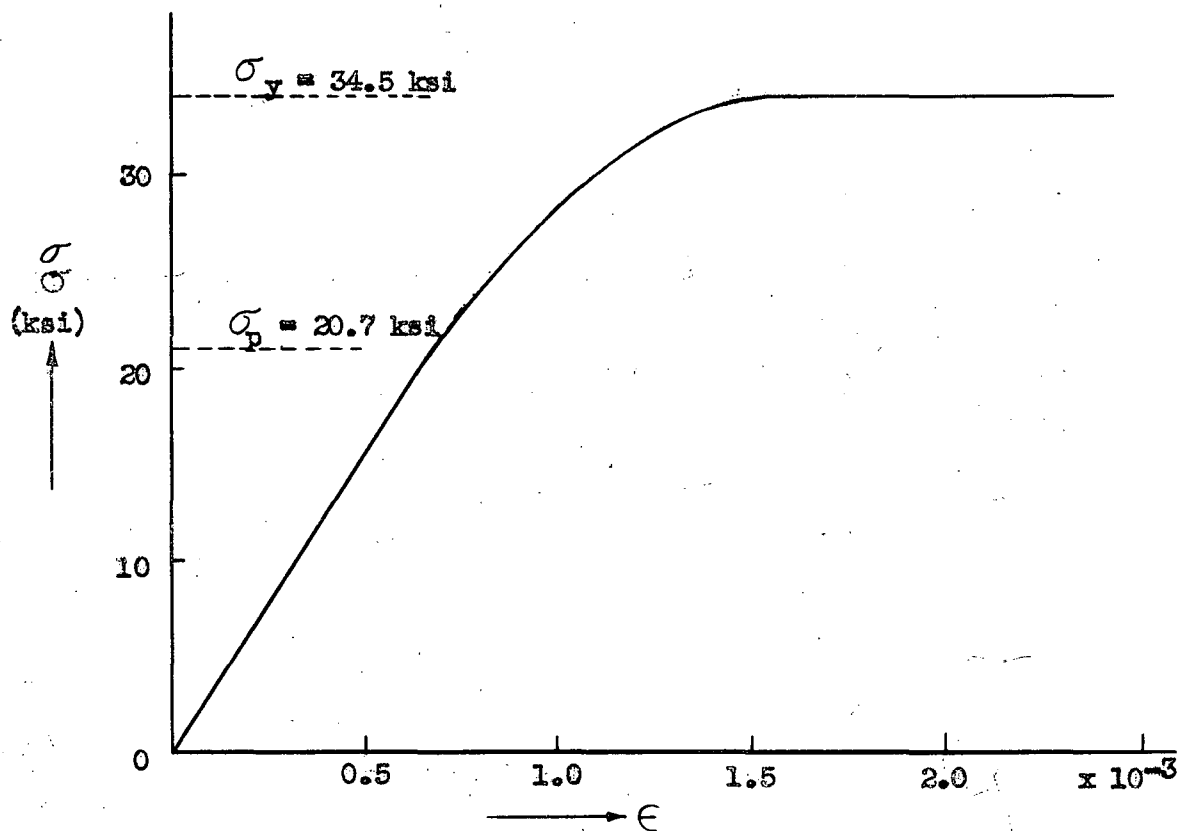


FIG. 18 AVERAGE σ - ϵ CURVE FOR STUB COLUMNS

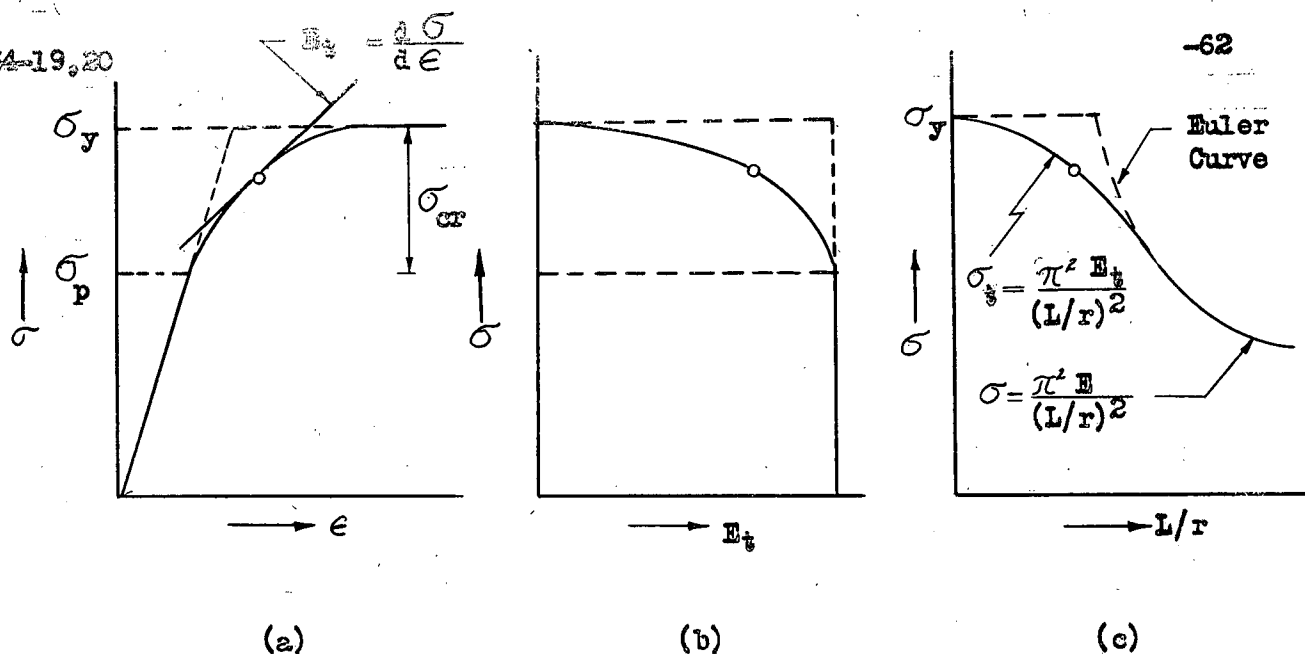


FIG. 19 INFLUENCE OF RESIDUAL STRESS ON THE "COLUMN CURVE" (IDEALIZED)

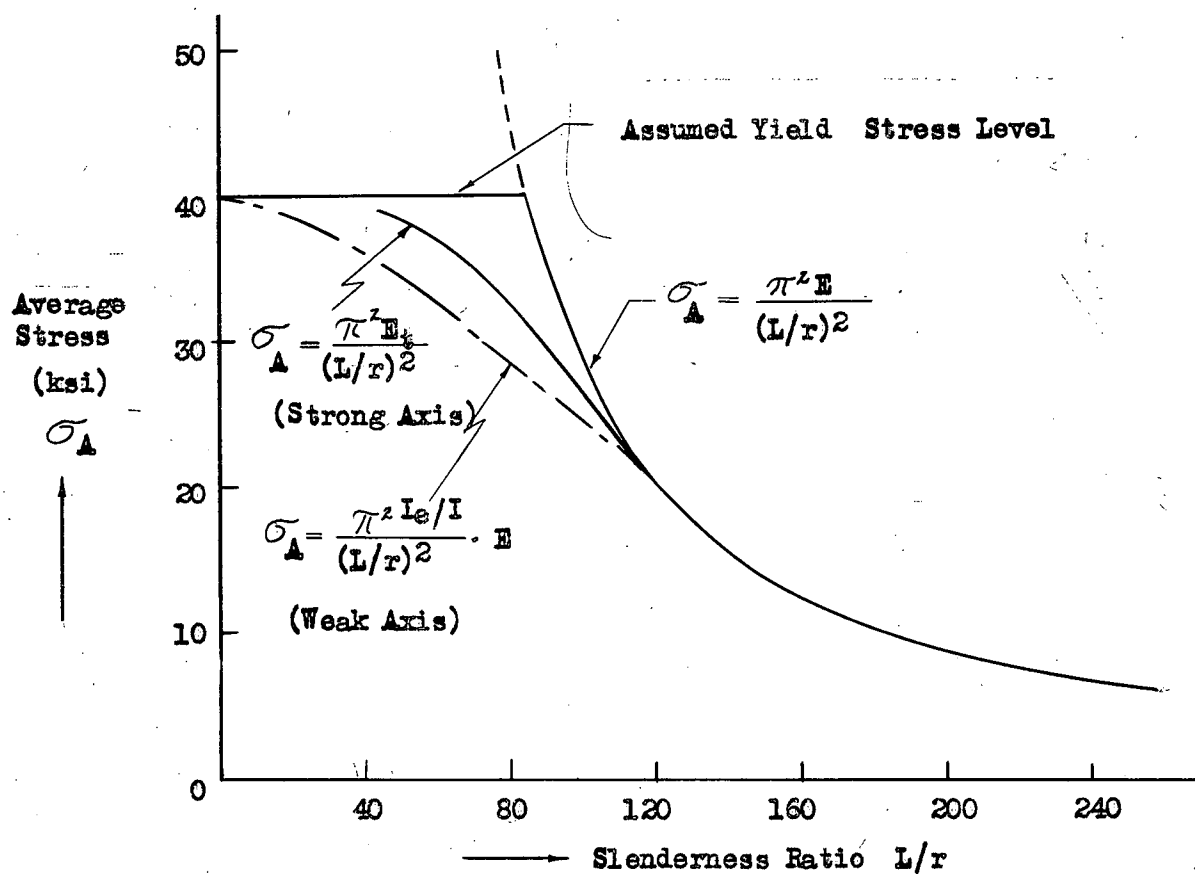


FIG. 20 IDEALIZED INFLUENCE OF RESIDUAL STRESS ON THE "COLUMN CURVE": EFFECT OF FLEXURE AXIS

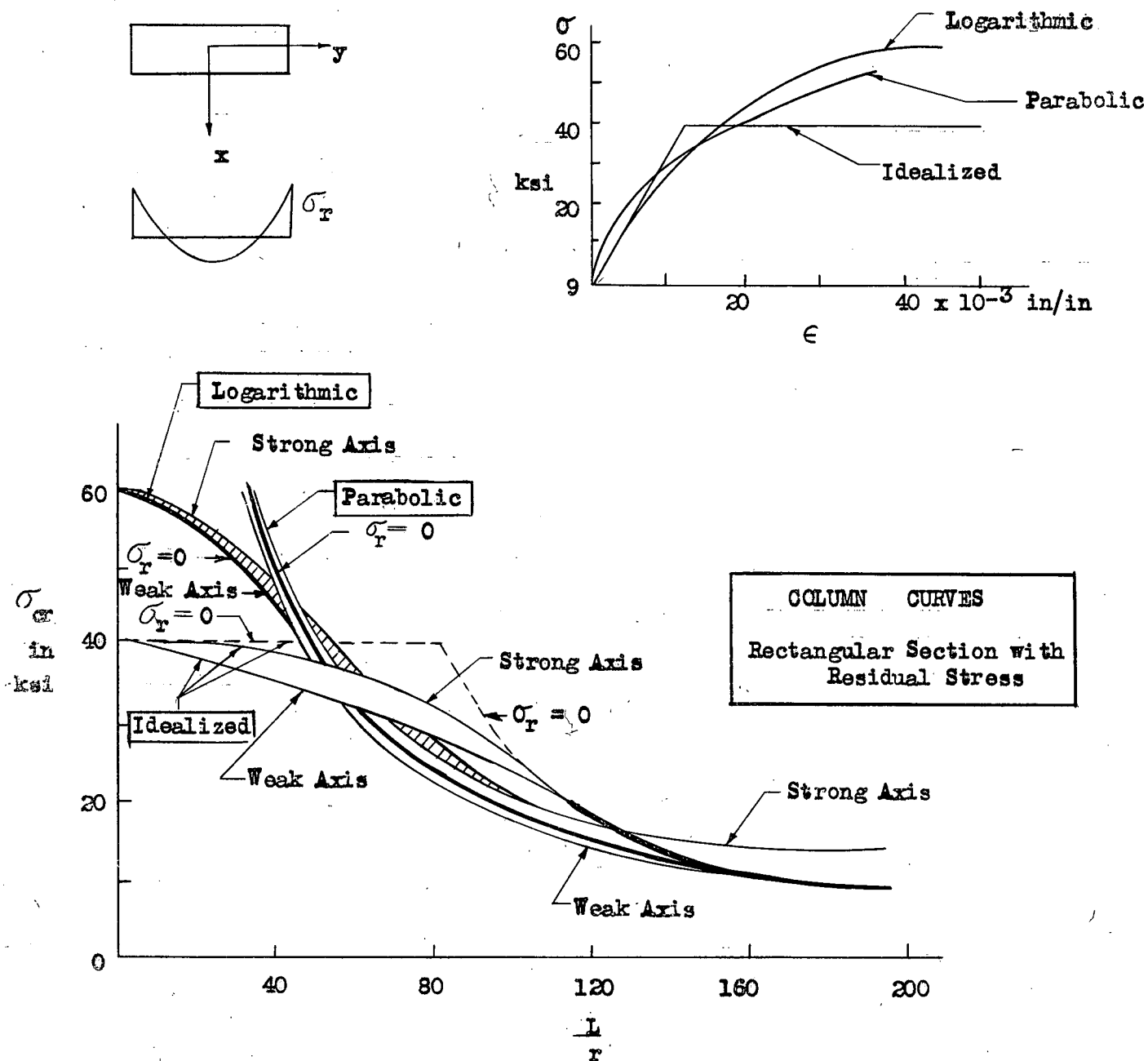


FIG. 21 STRESS STRAIN RELATIONSHIP AND COLUMN INSTABILITY

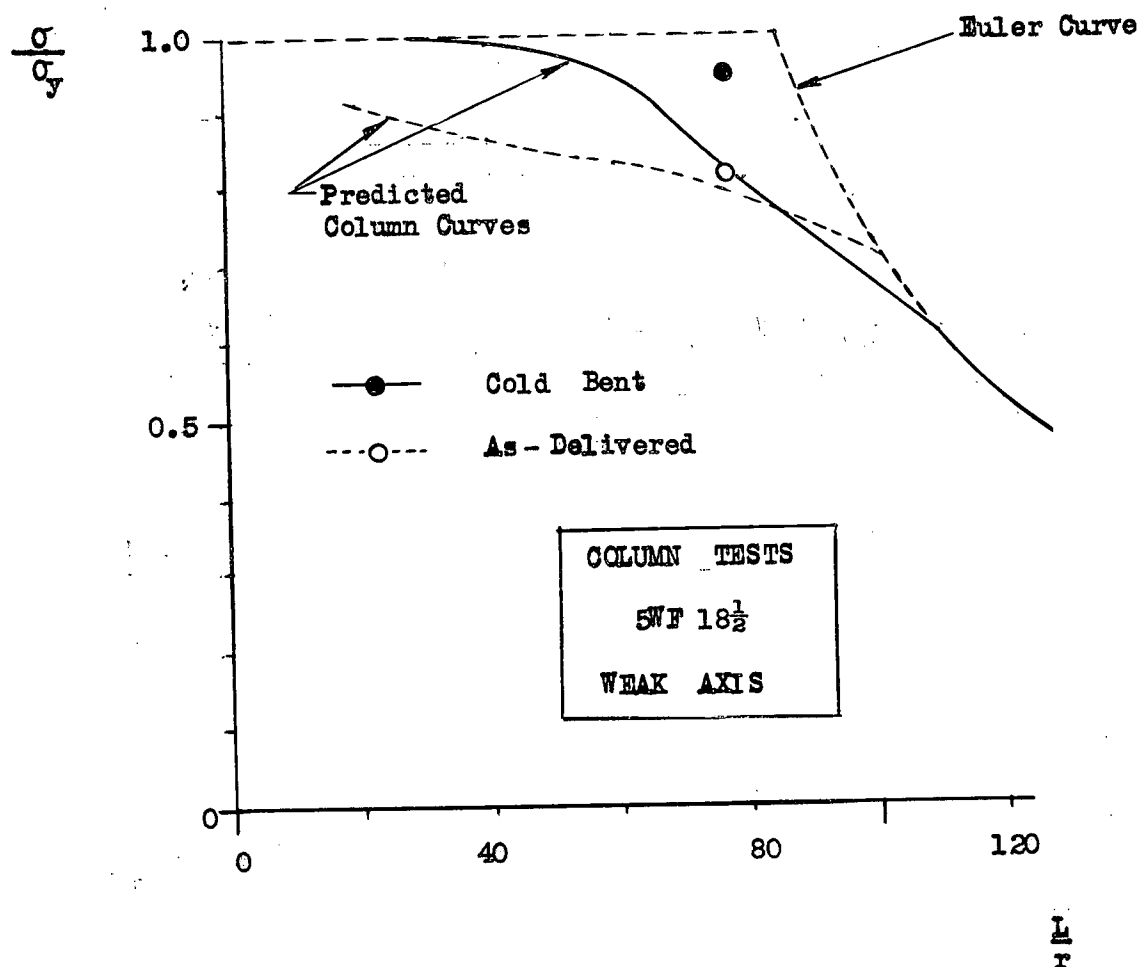


FIG. 22 COLD-BENT COLUMN —
COLUMN CURVE

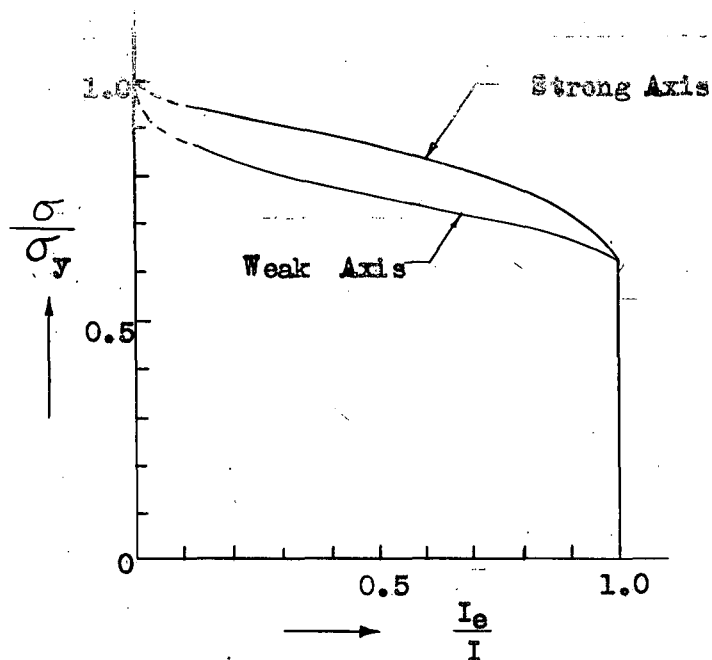


FIG. 23(a) MOMENT OF INERTIA REDUCTION FACTOR FOR WF COLUMNS

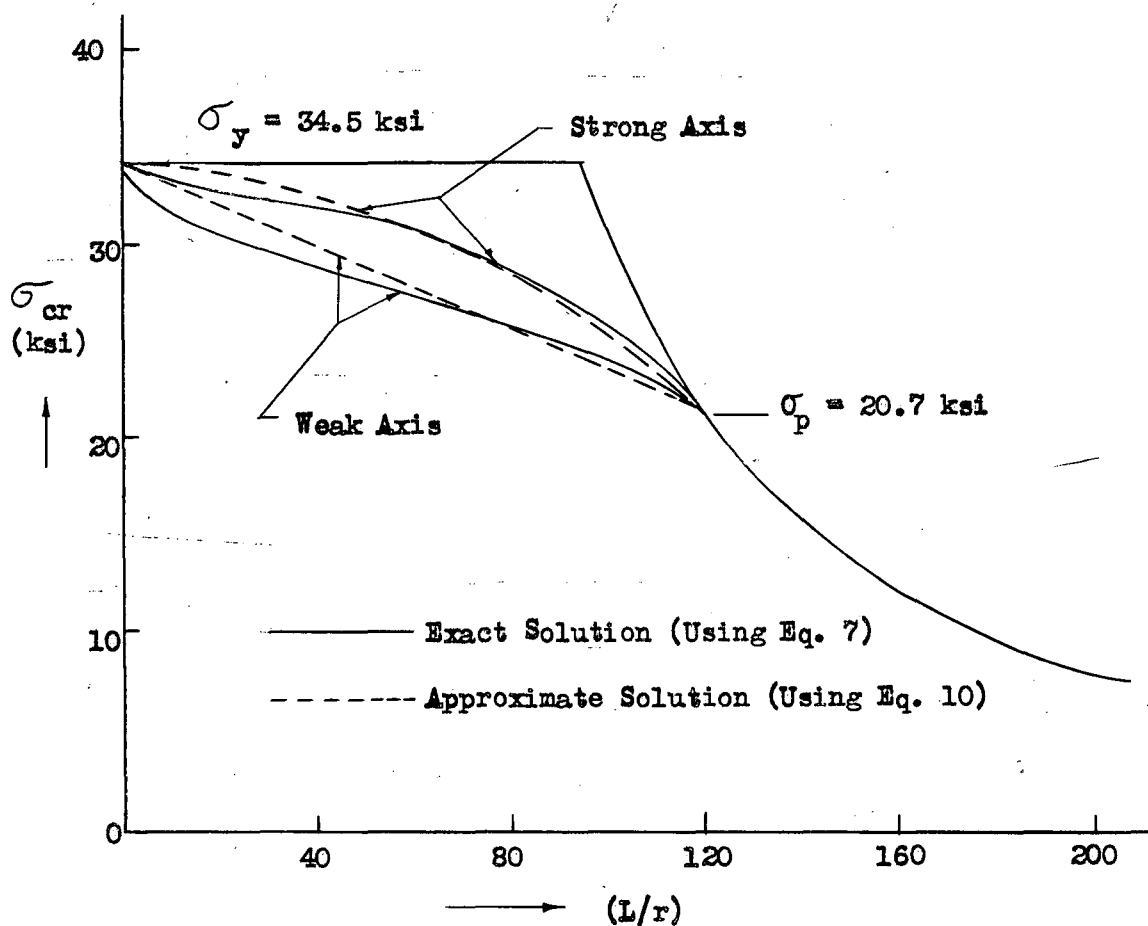


FIG. 23(b) COLUMN CURVE FOR WF COLUMN WITH COOLING RESIDUAL STRESS

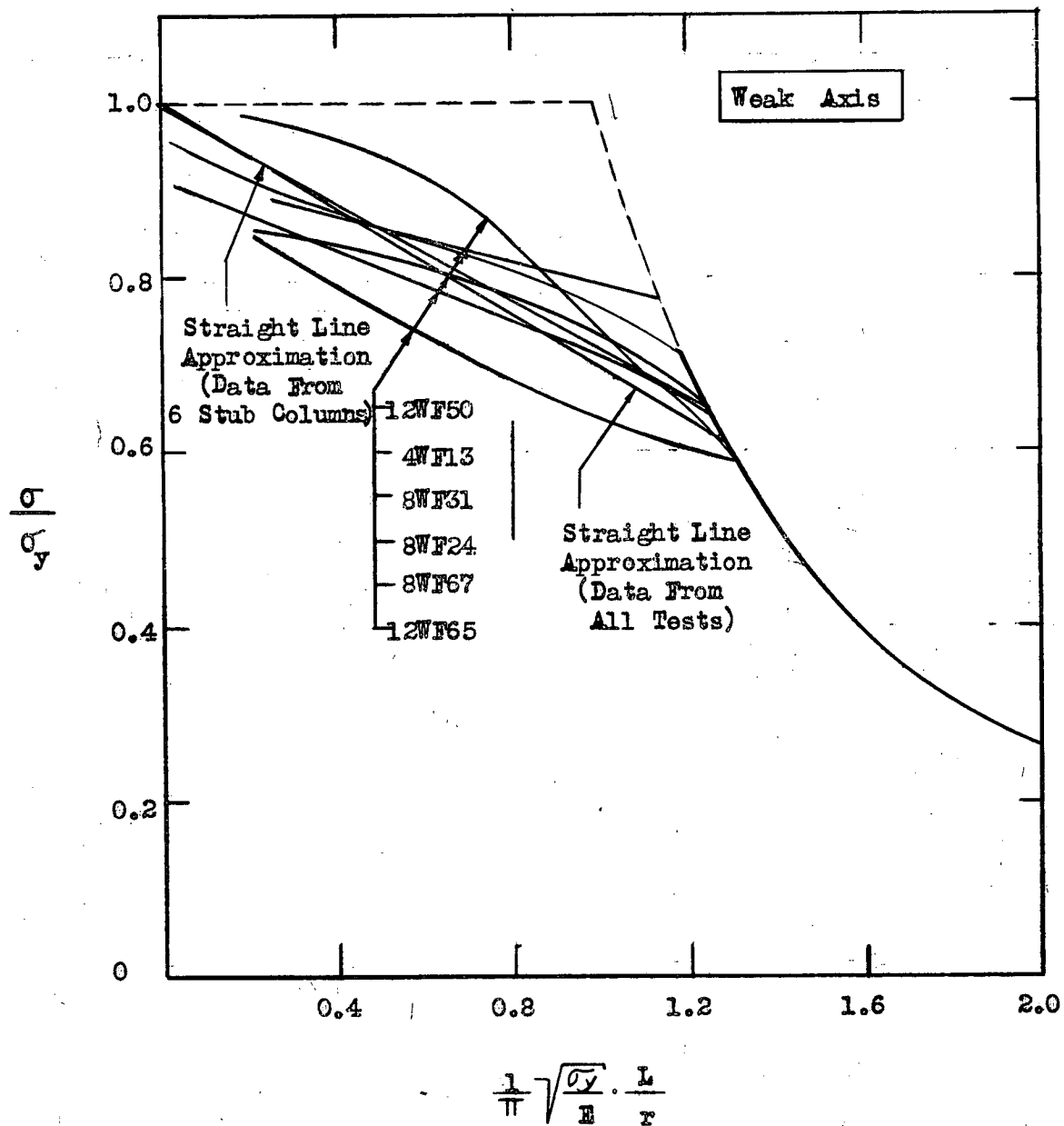


FIG. 24 COLUMN CURVES OBTAINED FROM STUB COLUMN TESTS

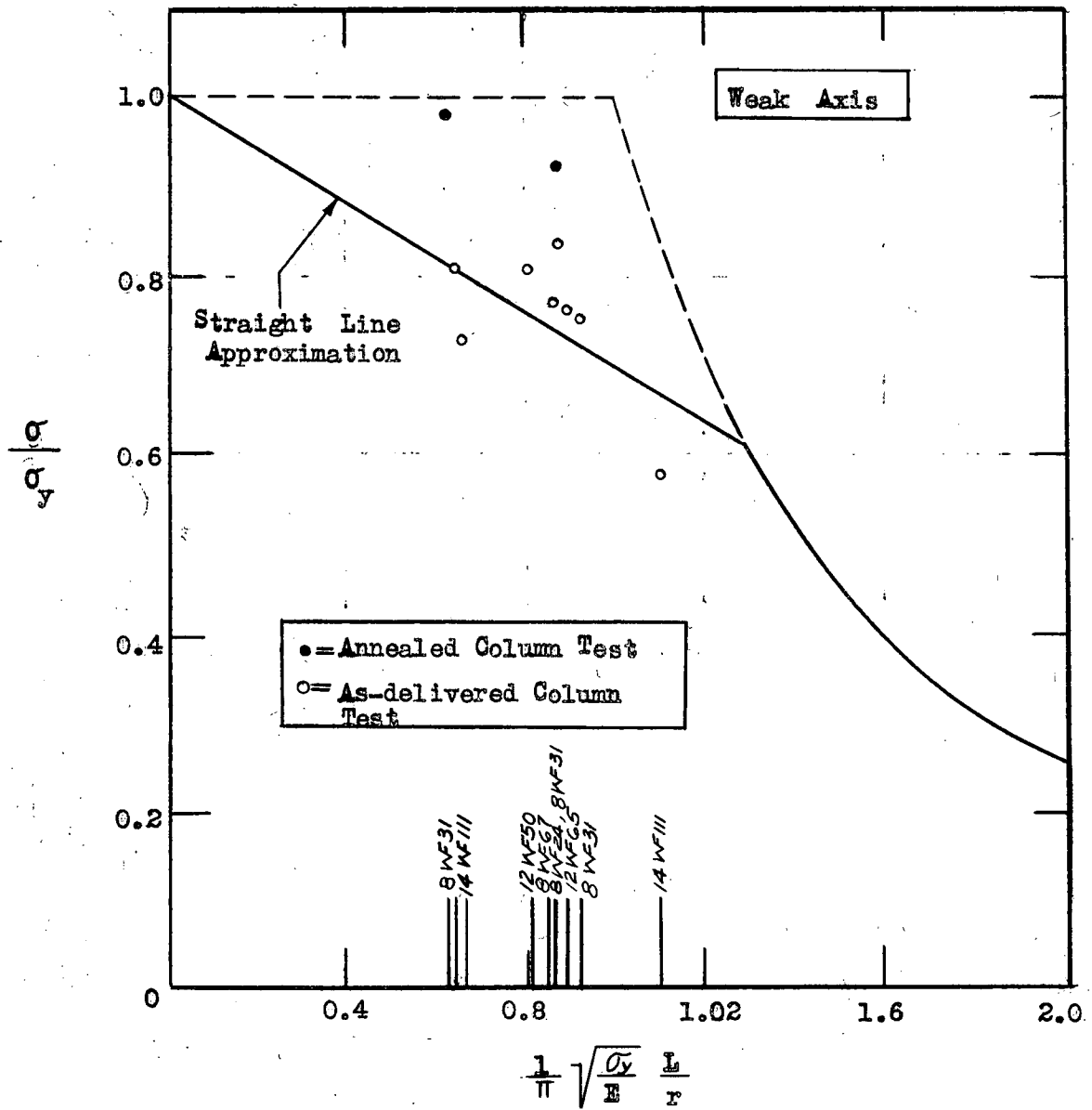


FIG. 25 COLUMN TEST RESULTS AND STRAIGHT-LINE COLUMN CURVE - WEAK AXIS

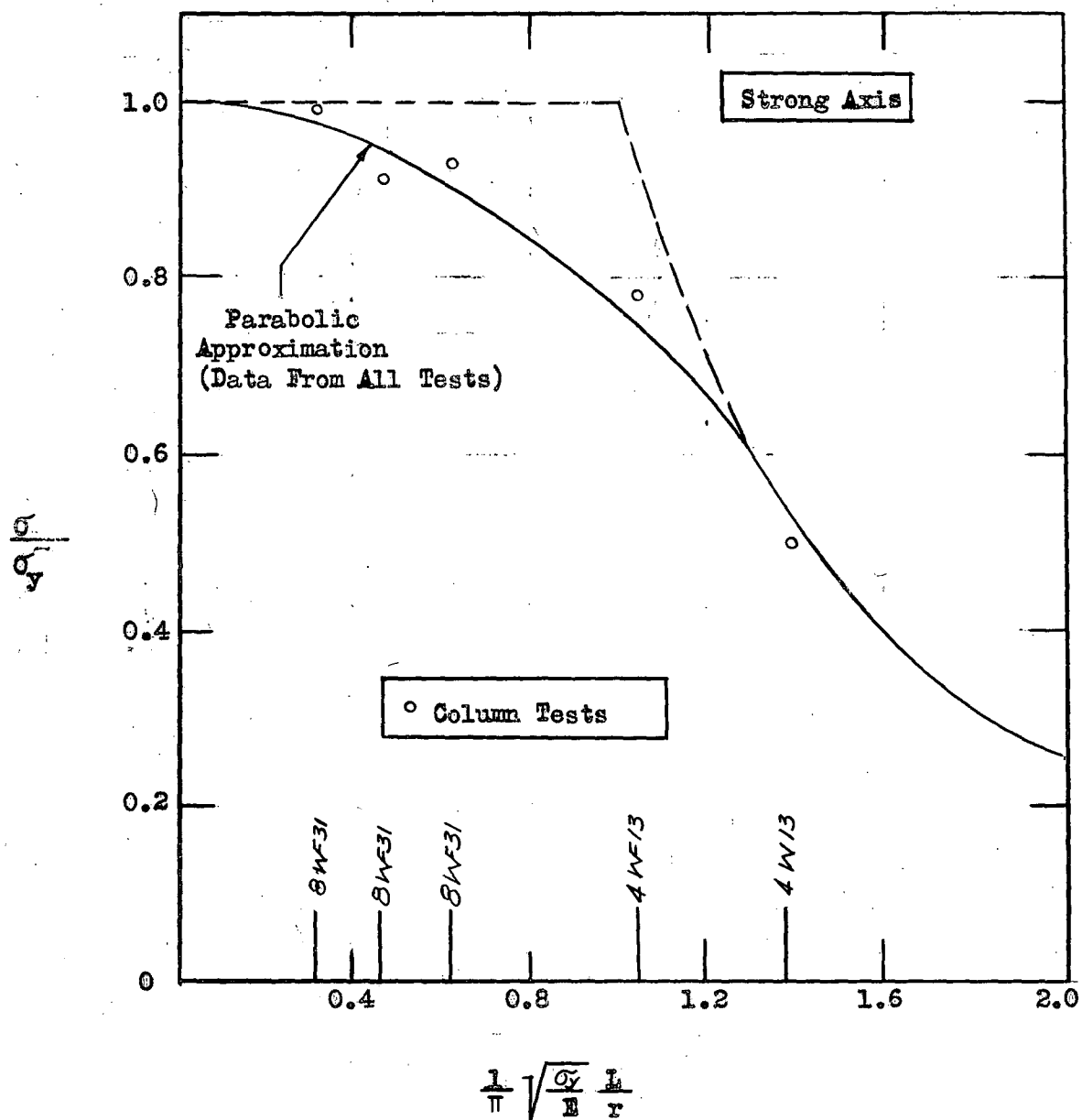


FIG. 26. COLUMN TEST RESULTS AND PARABOLIC COLUMN CURVE, STRONG AXIS

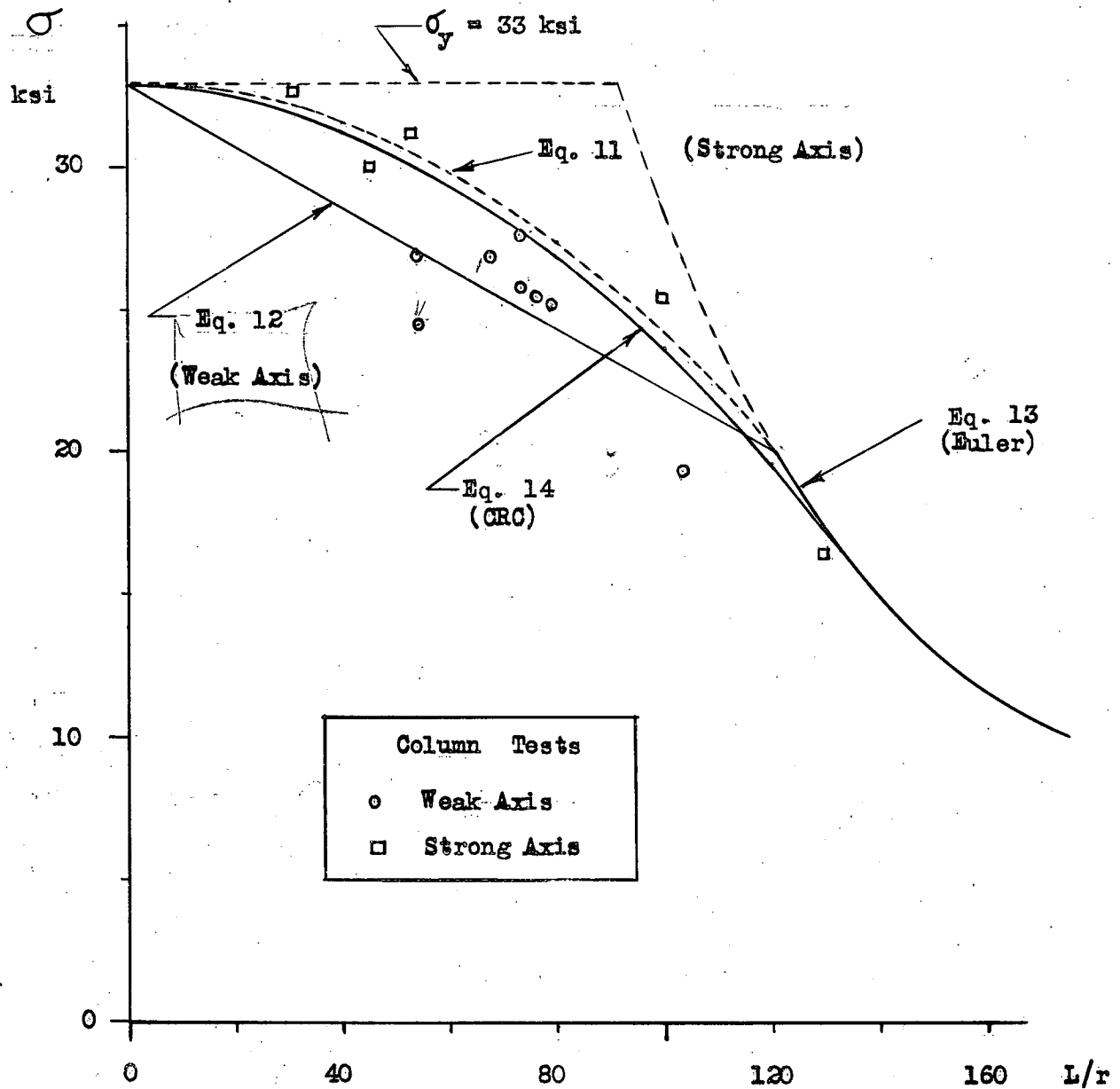


FIG. 27 BASIC COLUMN CURVES

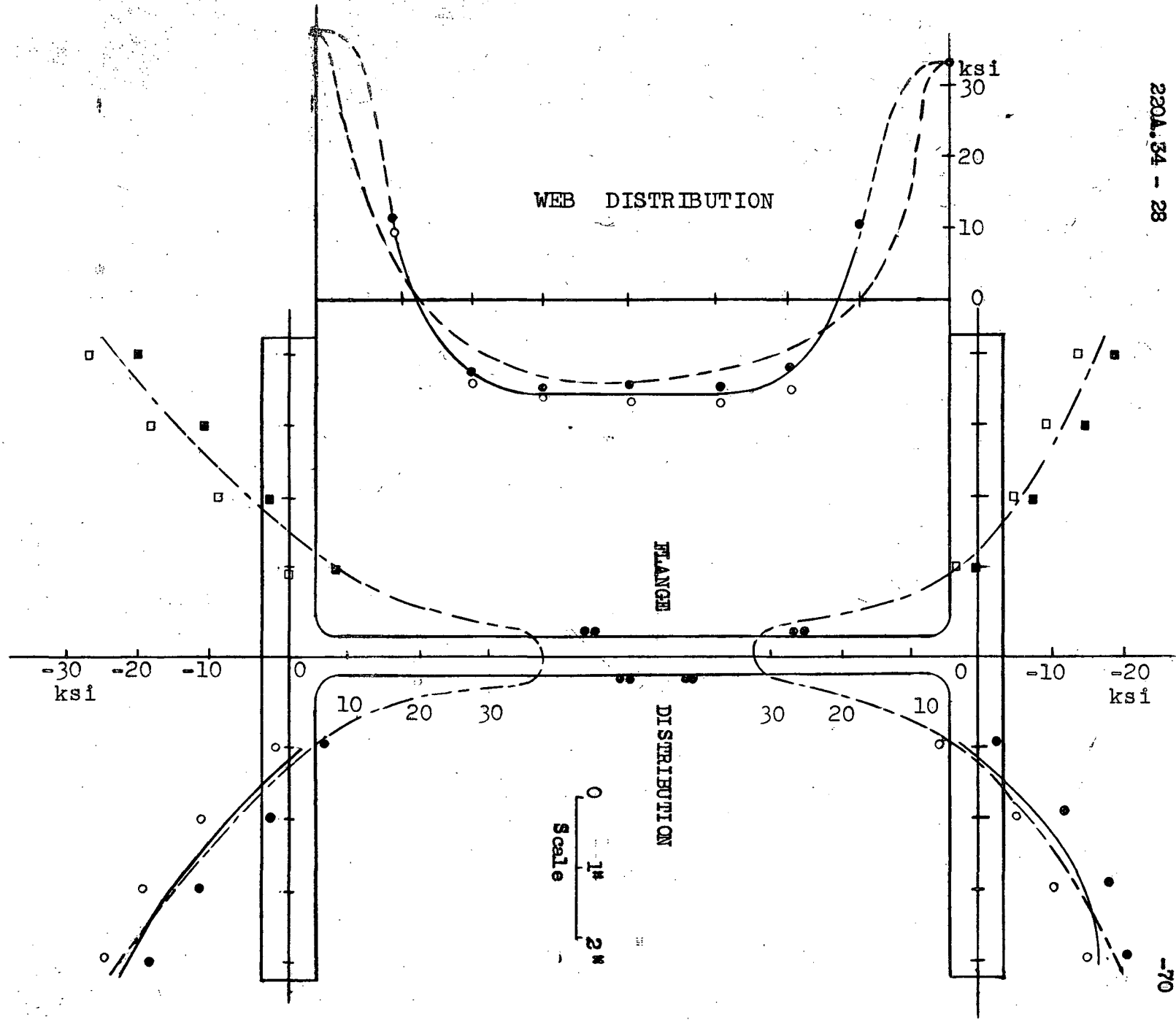


FIG. 28 RESIDUAL STRESS PATTERN IN WELDED MEMBERS

RESIDUAL STRESS PATTERNS

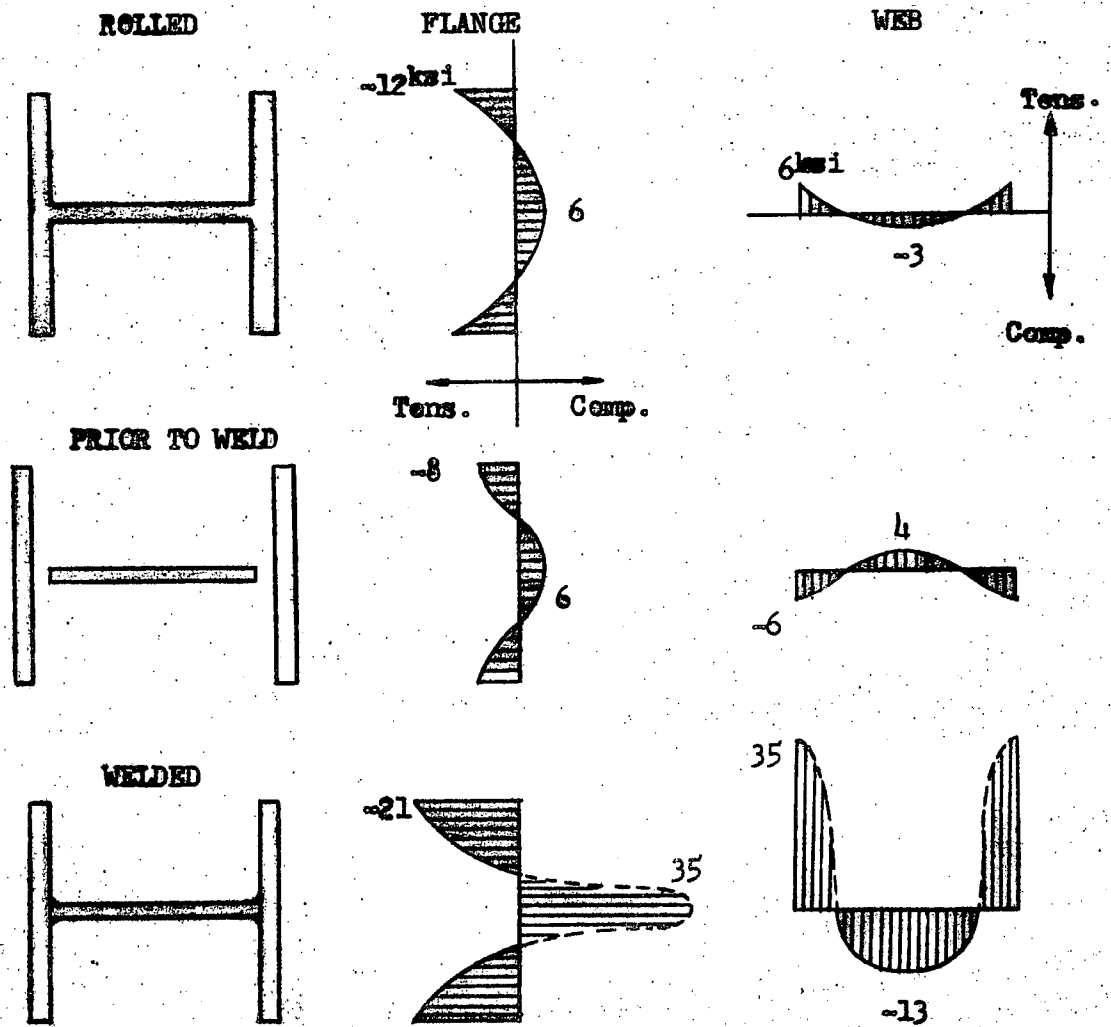


FIG. 29 RESIDUAL STRESS PATTERNS IN ROLLED MEMBER, IN PLATES, AND IN WELDED MEMBER

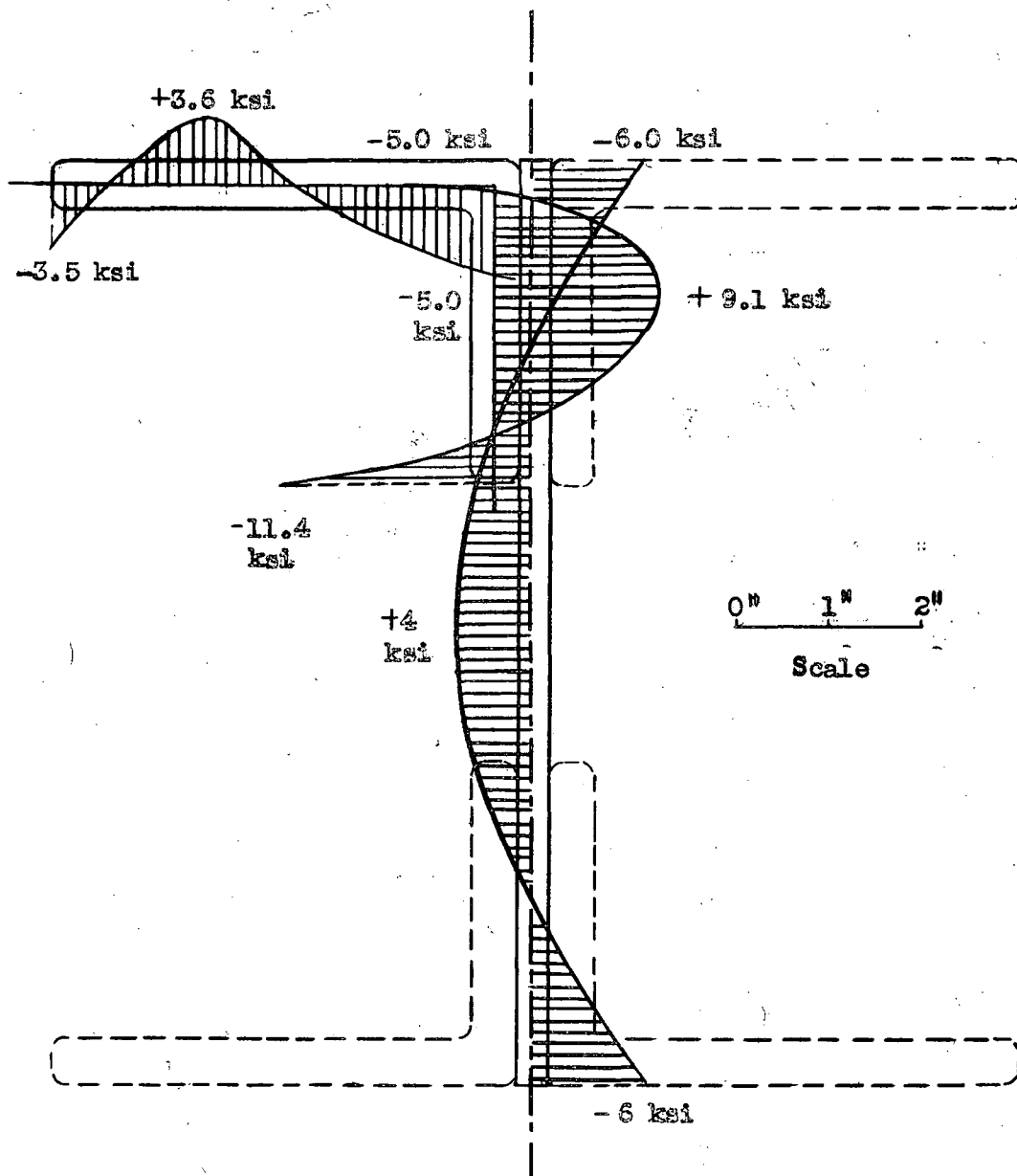


FIG.30 SIMPLIFIED RESIDUAL STRESS PATTERN FOR RIVETED COLUMNS

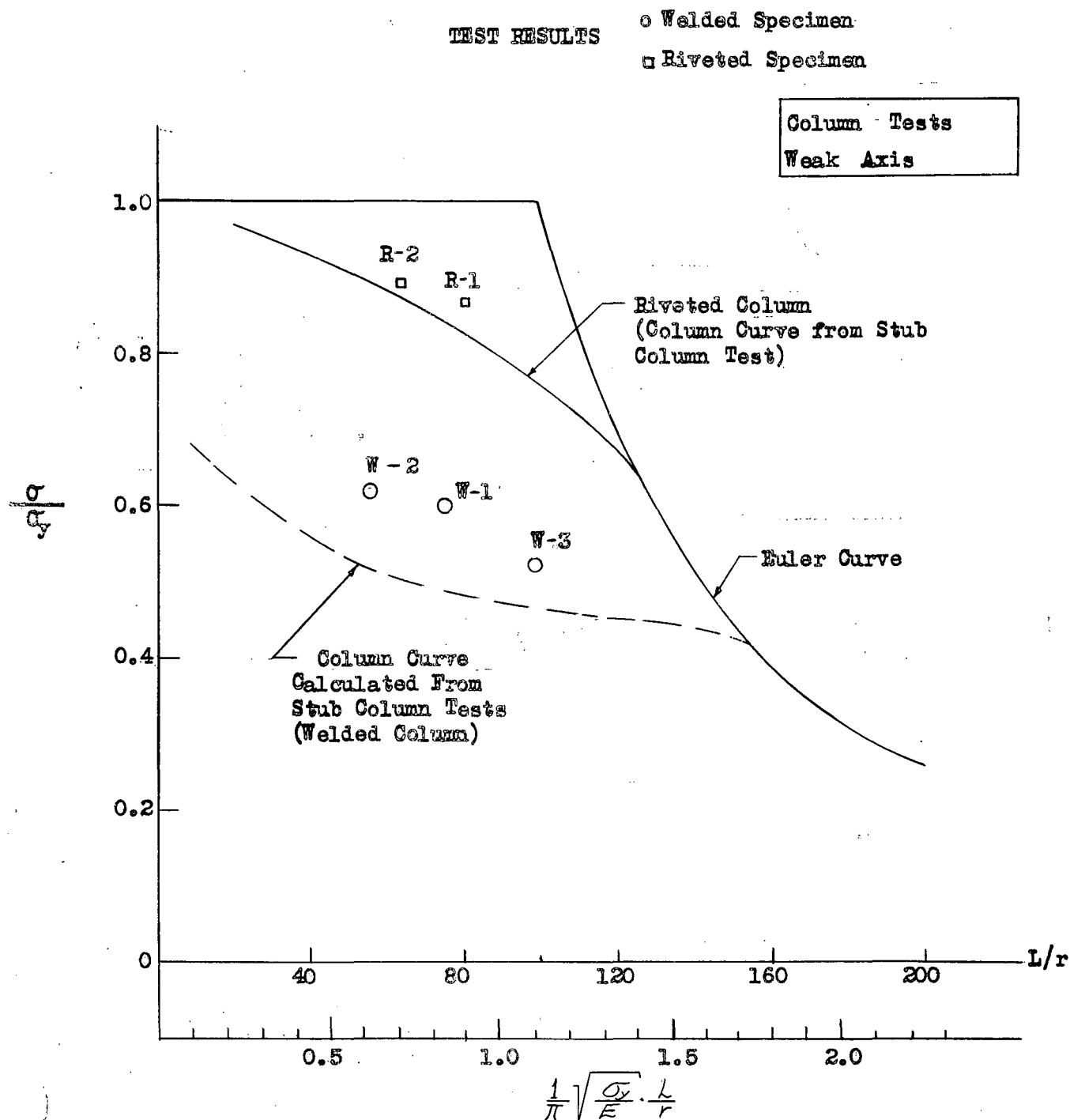


FIG. 31 TEST RESULTS AND COLUMN CURVES FOR WELDED AND RIVETED MEMBERS (WEAK AXIS)

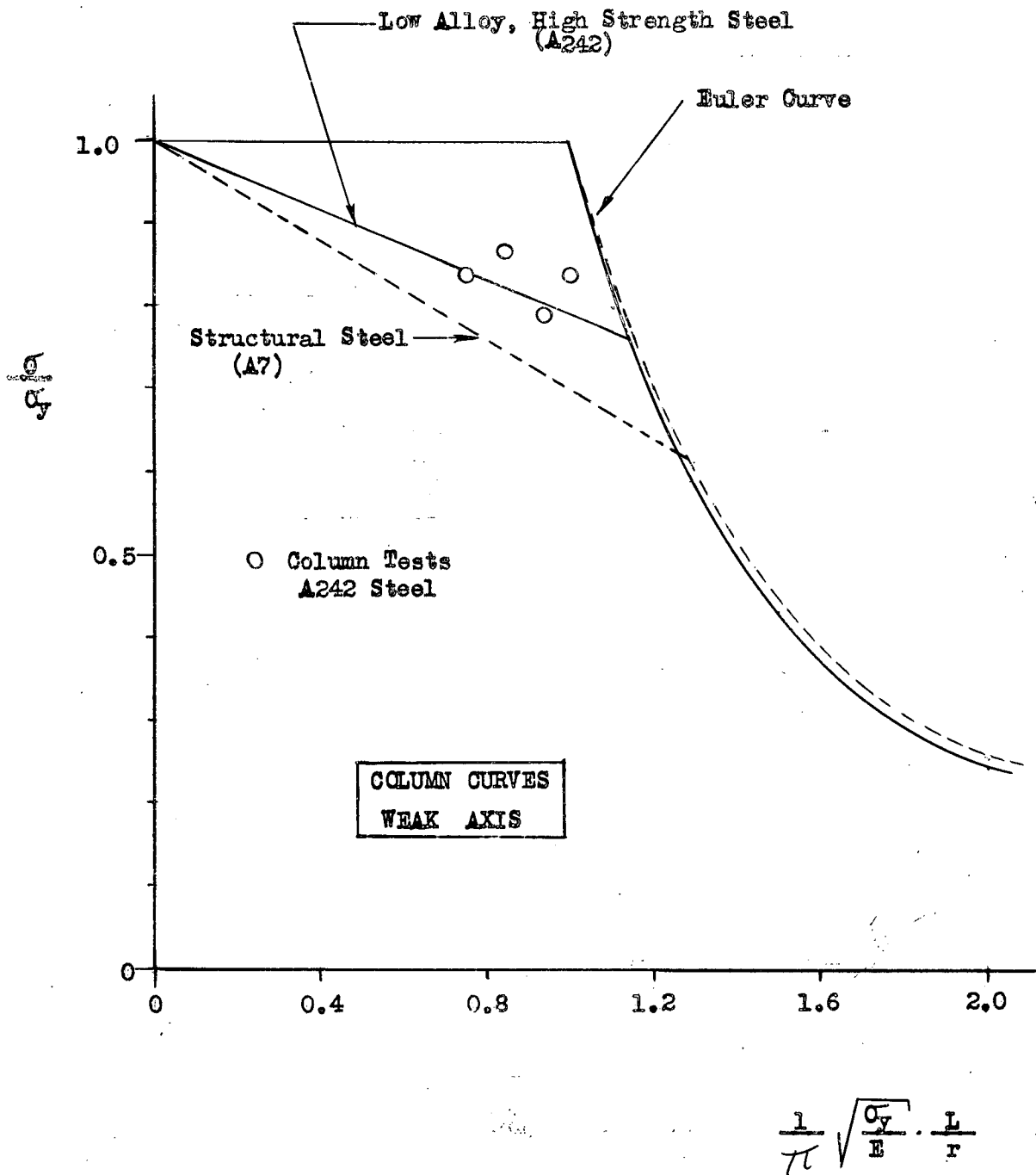


FIG. 32 RESIDUAL STRESS AND THE YIELD STRESS LEVEL

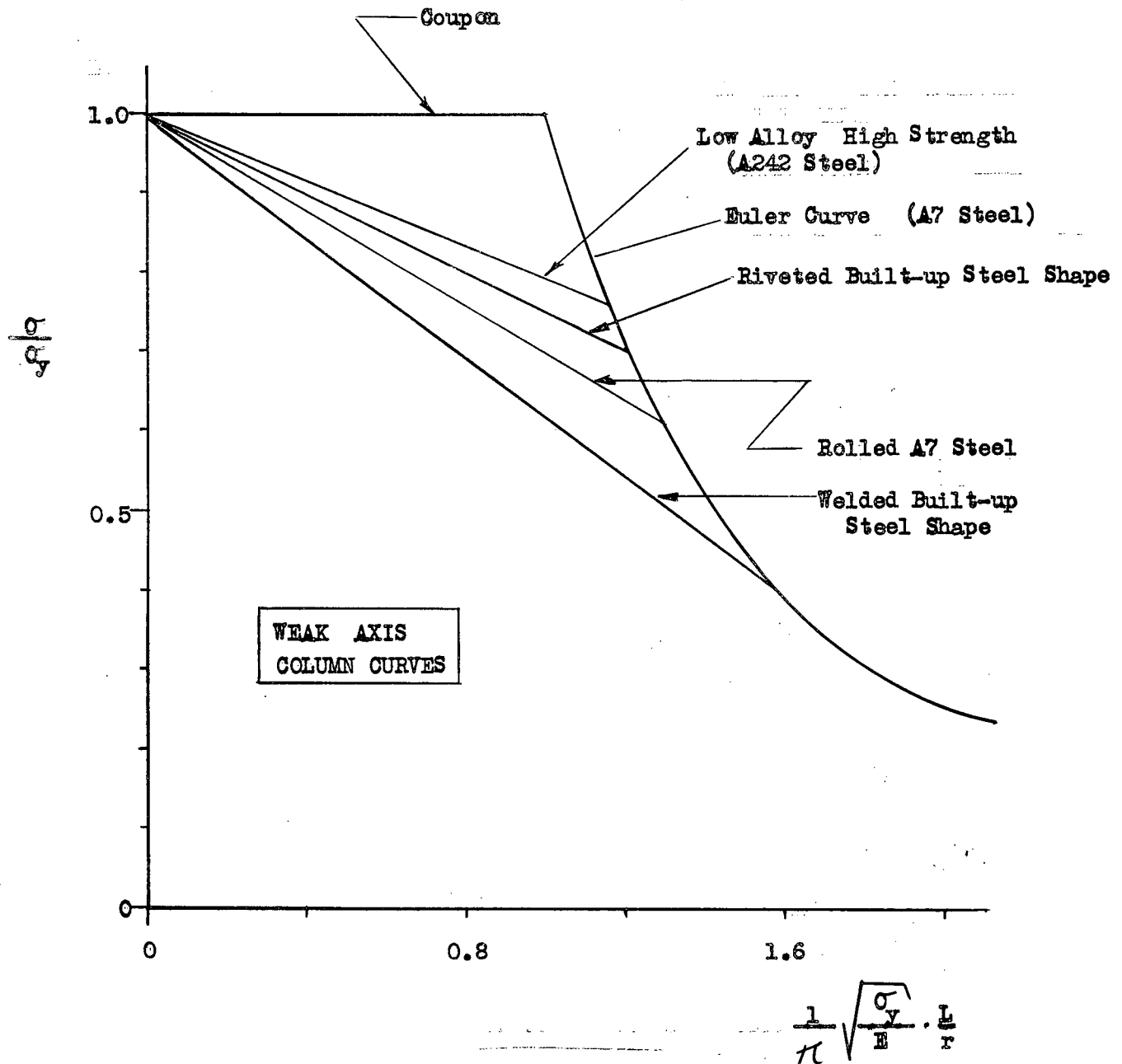


FIG. 33 COLUMN STRENGTH AS INDICATED BY THE MAGNITUDE
AND DISTRIBUTION OF RESIDUAL STRESS

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BASIC COLUMN STRENGTH

By the term, "Basic Column Strength" is meant the strength of a centrally-loaded straight column, with pin-ends

A research project on the behavior of such columns has been in progress at Lehigh University under the guidance of Research Committee A of the Column Research Council and with the further support of National Science Foundation, Penna. Dept. of Highways and the Bureau of Public Roads.

It is the purpose of this talk to summarize the findings of this investigation. We have been primarily concerned with rolled shapes A7 steel, although attention has also been given to welded and riveted built-up columns and to ~~some~~ low-alloy high-strength steel. This discussion is a preliminary to later papers that will discuss the behavior of members as they are actually found in a structure with restraints, eccentricities, and applied moments.

The CRC assigned to Committee A the task of determining the relationship between material properties and the strength of columns. The first pronouncement of the Council (based on the recommendation of Committee A) was its Technical Memo. No. 1, "The Basic Column Formula". This memorandum states that the critical or ultimate failure load of a column is given by the tangent modulus formula. This formula requires a knowledge of the stress-strain relationship, ^(that is, "material properties") and the first slide shows how it is applied.

(A)

Slide ~~(A)~~ (D)

Consider first an ideal coupon free from residual stress. Since E is constant up to the yield stress level, σ_y , the Euler formula would apply up to that point, and the tangent modulus concept would ~~be~~ not be relevant.

$E_t = E$ if $E_t = 0$.

Early work has shown that rolled or fabricated shapes do not behave like ideal coupons. Among other things, they contain residual stresses, and the ~~material~~ yield level may vary across the section. As a result the stress-strain curve ceases to be linear above a certain point.

To construct a column curve for such a material, first the tangent modulus E_t would be determined for various stress levels using the stress-strain diagram at the left. E_t is the slope of the curve. Then a stress-vs- E_t curve would be drawn as shown in the center. $E_t = E$ until the proportional limit is reached, after which it decreases to zero at ~~the~~ σ_y . Applying the tangent-modulus formula, the resulting column curve of stress-vs-slenderness ratio would be obtained as shown at the right.

The influence of ~~of residual stresses~~ ^{curving} when failure occurs in the inelastic range is thus seen by comparing the solid line with the dashed line obtained for a member without such stresses. In the elastic region residual stresses are of no influence.

Slide ____ (B)

This slide shows that a "rounded" stress-strain diagram is not a figment of the imagination but is a reality. Stress is plotted against strain for an ideal coupon (dashed) and for a "stub column" (solid). 18WF 105

~~As mentioned earlier, two factors exist~~

The column curve depends upon the stress-strain relationship. The latter, in turn, is dependent upon two important factors:

these are: (a) the magnitude and distribution of residual stresses (which cause a lowering of the proportional limit and affect the shape of the curve above $\bar{\sigma}_p$), and (b) the basic yield stress level of the material practical (which affects the/upper limit of column strength)

These two variables will now be examined.

Residual stresses are formed in a structural member as a result of plastic deformations. In rolled ~~or hot rolled~~ shapes these deformations always occur during the process of cooling from the rolling temperature to air temperature; the plastic deformations result from the fact that some parts of the shape cool much more rapidly than others, causing inelastic deformations in the ~~rolling~~ slower cooling portions.

If there is no plastic deformation there are no residual stresses.

Slide _____ (C) F

This slide shows a ^{rolled} shape during one of the last passes. Below is the end of a previous ~~rolling~~ rolling. As cooling continues, the tips get black, hard, and resist the contraction of the hot portion, causing plastic deformations there.

Slide _____ (C) F

Use (G)

another
Here ~~the~~ beam has cooled ~~another~~ to the point that the web is quite dark while the ~~flange~~ flange-web juncture is quite hot.

Slide _____ (C) H

At a later stage on the cooling bed the flange tips are black, while the web-flange juncture is red. When this shape cools to room temperature we might guess that the red parts would remain in residual tension and the black parts in compression.

Slide _____ ~~(2)~~ I

This slide of "typical" residual stress patterns shows that our guess was rather good. In fact, in hot-rolled or welded members, the part to cool last is usually in a state of tensile residual stress.

Of the many sets of residual measurements that have been made, this slide shows the results ~~of these~~ ^{for} three shapes of widely differing size and geometry. While the variation is considerable, the general pattern in the flange is similar. Insofar as columns are concerned, we will see later that the most important of the stresses are those at the flange tips. The average compressive stress measured there is about 13,000 psi.

With these typical patterns, we are now ready to examine ^{clearly} ~~note~~ their influence upon the apparent stress-strain relationship.

Slide _____ (F1) J1

The cutting of a coupon from the flange of a member would relieve the residual stresses that were present prior to sectioning. Thus the stress-strain diagram would be as shown by the dashed line in sketch (b).

Slide _____ J2
(12)

If, now, the load is considered as being applied to the entire cross section containing its residual stresses it is evident that ^{the behavior will be linear until} when the applied stress becomes equal to the difference between σ_y and σ_{rc} . Then yielding will commence at the flange tips.

A linear distribution of residual stress has been indicated. As shown in sketch (b) the stress-strain curve will remain linear so long as the applied stress is less than σ_p .

J3
Slide _____ (P5)

When more load is applied, the average stress and average strain are no longer proportional to one another because of yielding of the flange tips. Thus a non-linear stress-strain relationship results for the section as a whole (red). The circle in sketch (b) corresponds to distribution (d).

After ~~the~~ yielding has penetrated across the entire section (blue) the stress distribution is identical to that of a shape containing no residual stresses. In effect they are "wiped out" and have no influence on the yield stress level.

07 ~~From sketch (b)~~ The "red" portion of the stress-strain diagram in sketch (b), then, reflects the influence of residual stress. It causes a marked reduction in the ~~tangent modulus when this stress is exceeded~~
(to about 20 ksi)
proportional limit and a consequent reduction in the tangent modulus value when this stress is exceeded.

The second important factor influencing the stress-strain relationship is ~~xx~~ the yield stress level.

Slide _____ (G)

This slide shows the different values of the yield stress level that may be obtained depending upon the type of test that is performed.

Starting with curve A, the yield value reported in a mill-type acceptance test is usually the upper yield point. Occasionally specimens will not exhibit an upper yield point (curve B) in which case the yield stress is usually recorded at a strain of 0.5%. The upper yield point is from 0 to 10% higher than the yield stress level.

Comparing curves A and C, the mill test will usually show a high^f level primarily because of the effect of strain rate. ~~Since~~ The plastic strain rate in a mill test is about 1000 micro-in/in/sec which results in a yield stress level for a web coupon that is from 10% to 15% higher than the "static" value (measured at zero strain rate).

The web of a WF shape is usually stronger than the (see curve D) flange (4 to 7%). A web coupon is ~~usually~~ used in the mill acceptance test because of ~~the~~ ^{its} parallel sides.

Finally it is seen from this slide that a stub column tested at "zero strain~~x~~ rate" ^{not only} reflects the effect of residual stress upon the stress-strain curve ^{but it also} ~~and~~ averages out the differences between the web and flange -- curve F.

Dotted curve E shows that weighted average of values obtained in tension tests of web and flange coupons agrees well with stub.

Slide _____ (G) (Cont'd)

It is evident, then, that the difference between an acceptance test and the basic compressive strength is about 20%. It is due mainly to the strain-rate effect, but is also influenced by the higher strength of the web and the difference between upper and lower yield points.

There is no need to be alarmed about this situation. It has always existed and has been reported in the literature as early as 1931.

In fact, this comparison is an important one for establishing a value for the yield stress level to use in a basic column formula. The relationship has now been established for a representative sampling and can next be applied to ^a ~~the~~ larger body of acceptance test data available ~~from mill tests~~ in the mills. The average of about 6000 mill tests ~~gives about 42 ksi~~ ~~tax~~ gives about 42 ksi (point to slide). Applying a 20% reduction gives 34 ksi -- a value which (coincidentally and conveniently!!) is close to the ~~spp~~ specification minimum of 33 ksi. It is also the average obtained in our tests

Slide _____ (I)

This is a picture of a typical stub column test.
 It is a 14 WF 426 shape being compressed in the 5-million-pound testing machine at Lehigh. The ~~maximum~~ load at the ~~the~~ yield stress level ~~was~~ of 33 ksi would be about 4-million lbs. Flaking of mill scale along the flange tips shows clearly the yielding that occurred there due to combination of loading stresses and the compressive residual stresses.

The program of tests ~~included~~ started with a 4WF31, included such shapes as 12WF65 and 14WF111, and on up to the heaviest rolled shape -- 14WF426.

Having evaluated the influence of these two important factors, we are now in a position to look more closely at the strength of H-shaped columns.

115
 33
 375
 375
 4125

(A)

Slide _____ (II)

Returning to the first slide, the tangent-modulus formula in which E_t is determined directly from the stress-strain curve is "precise" only for the special case of a rectangle bent about the weak axis.

It is a good approximation to the strength of a WF column bent about the strong axis and the weak axis strength may be expressed in terms of this same value.

Without going into detail, the real key to the solution lies in a consideration of the moment of inertia of the yielded cross-section.

Slide _____ (J) (Cont'd)

This tendency is also shown in this slide. The lower curves are for flexure about the ~~weak axis~~ of an H-section, while the upper curve is for flexure about the strong axis.

The dashed lines in each case represent so-called "exact" solutions. It will be noted that the curve for buckling about the strong axis is approximately parabolic in shape and that for buckling in the weak direction the curve may be approximated by a straight line.

In fact, all that is needed to construct these curves is to determine the yield stress level (shown at about 34 ksi) and the flange-tip compressive residual stress. (Subtracting from 34ksi the average measured value of 13 ksi gives a proportional limit of about 21 ksi as shown).

The correlation of these theoretical considerations with actual column tests will now be shown.

Slide _____ (~~K~~) N

This slide shows the results of weak-axis column tests in comparison with the same straight-line approximation previously. The circles show the maximum load the columns carried. The coordinates are non-dimensionalized in order that variation in E and σ_y could be eliminated in the comparison. However, they still remain as functions of σ_y and $\frac{L}{r}$.

The results of annealed columns are also shown by the solid dots. Their strength is clearly well above that of the as-delivered members.

Slide _____ (E) O

This slide, similarly, shows the results of column tests (strong axis) and the parabolic approximation of the earlier slide. Again there is good correlation between theory and tests.

Slide _____(M)

This is a photograph of one of the columns under test in the 5-million pound machine at Lehigh. It is a 14WF111 shape with L/r of about 100.

Up to this point our theory has neglected one factor that is important for very short columns ---namely, strain-hardening.

Slide _____ (M1)

As shown in this stress-strain diagram, structural steel strain-hardens after the yield plateau has extended, about 10 times the elastic limit value *plastic strain reached* as shown in the first part of 5 M1. (X1)

This results in a σ vs E_t diagram that *differs from Fig 19.* ~~is somewhat~~ different than before. Instead of $E_t = 0$ at the yield stress, $E_t \approx 900$ ksi, or about one-fortieth the value of E . (B)

The influence upon the stress-vs L/r curve is as shown. Below $L/r \approx 20$ a column will carry a greater average critical stress than the full yield value. (X2)

The slide which follows has been constructed ~~making~~ taking into account the strain-hardening influence upon short columns.

Slide 220A.44 (N)

This is a summary of the work on rolled WF shapes of A7 steel. All tests have been adjusted to a yield of 33 ksi and an E of 30,000 ksi. The point of strain-hardening is indicated at $L/r=20$.

The weak-axis curve (straight line) is shown in red

The strong-axis curve (parabola) is shown in green.

In order to simplify the design procedure by eliminating the consideration of flexure axis, Dr. Johnston ^{Chairman of CFC} has suggested that the proportional limit be selected as one-half the yield level and that it be used in conjunction with a parabolic curve. ~~As a single curve would be used for both strong and weak axis bending.~~ This single curve, shown in blue, would be used for both strong and weak axis bending. It is a fair approximation to both of the approximate curves and to the test results.

Some pilot tests have been performed on built-up columns, both of the welded type and those fabricated by riveting.

Slide 200A.45 ~~(#)~~ (U)

This slide shows a comparison of residual stresses in WF shapes, in universal plates prior to fabrication, and in welded and riveted sections.

The as-rolled plates contain significant residual stresses due to cooling after rolling.

The welding introduces high tensile residual stresses at the flange-web juncture (they approach the yield value) and this gives rise to ~~xxxx~~ compressive stresses that are higher than those encountered for the rolled shapes -- at least for these tests. $9 \times 3\frac{3}{4}$ plates were ^{scabed} ~~used~~ with $\frac{5}{16}$ " ~~web~~ fillet welds.

The riveted shape has only those stresses that were present in the angles and plates prior to fabrication -- and these stresses are rather low.

A

$9 \times 3\frac{3}{4}$ pl
4
 $\frac{5}{16}$ " web

Slide _____ ~~(a)~~ (v)

Both theory and tests confirm the results that would be guessed on the basis of measured residual stresses.

The riveted columns with low stresses exhibited relatively higher column strength.

The welded columns with higher compressive residual stresses at flange tips gave lower strength. In this connection, the magnitude and distribution of welding residual stresses are markedly influenced by the geometry. Further work ^{was} is therefore required on members with cross-sectional shapes other than the H-section. It ^{was} would be expected, ^{for example} that the use of welded H-shape columns would be replaced more frequently by "box" sections and these would undoubtedly show a higher strength.

Slide _____ (R)

A program of tests on high-strength low-alloy steel has indicated that the column strength of such steel can be predicted in the same manner as for A7 steel. These results are shown non-dimensionally in this slide to afford a comparison with A7 steel.

These studies have shown that the magnitude and distribution is about the same in the two grades of steel. Therefore, because of the high yield stress level (55 ksi), the influence of residual stress on columns of high strength steel is not as pronounced as on A7 steel.

Slide _____ (S)

This final slide gives a comparison and summary of the strength of pin-ended centrally-loaded columns.

1. Residual stress primarily affects the proportional limit. $\sigma_p = \sigma_y - \sigma_{rc}$

2. The yield stress level represents the upper limit of column strength.

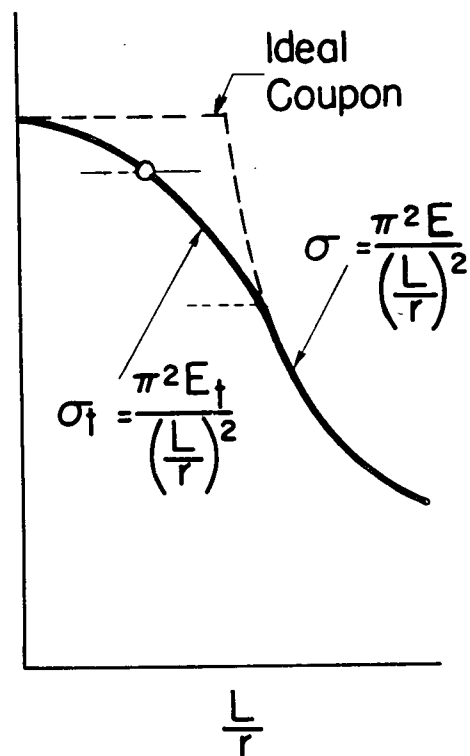
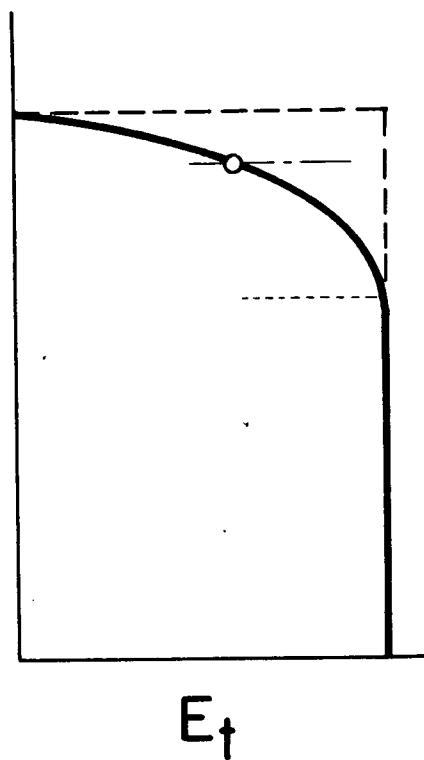
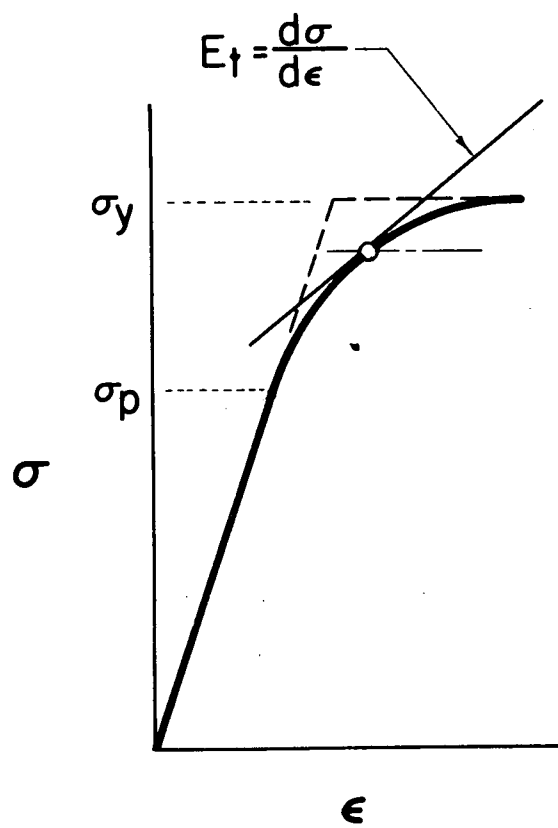
3. Residual stresses in high-strength low-alloy steel are about the same as in A7 steel. Thus the percentage reduction in column strength due to this factor is less for the stronger material.

4. A straight line is a good approximation to the behavior of A7 rolled WF shapes bent about the weak axis.

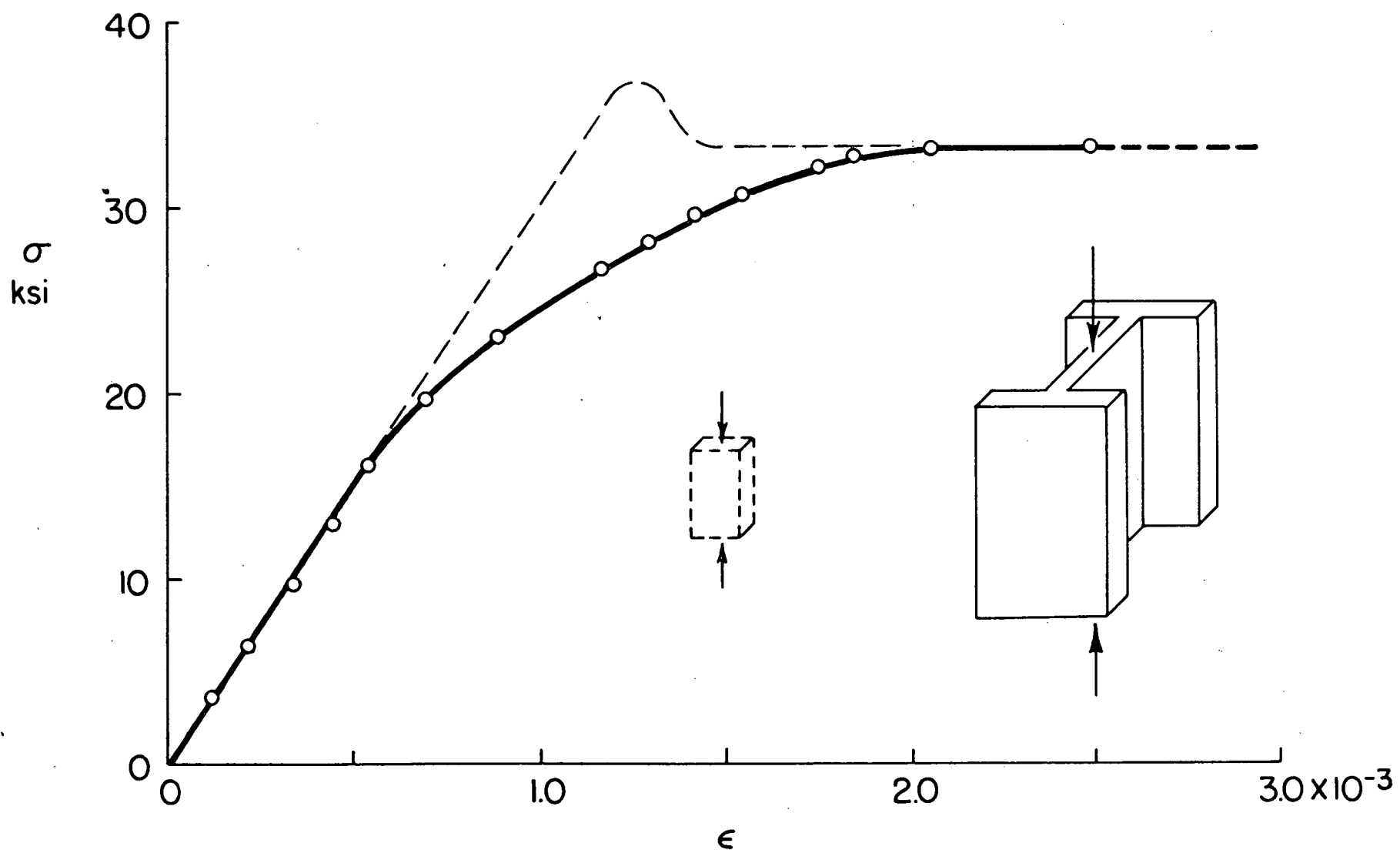
5. Riveted H-shaped columns contained variable residual stresses of relatively low magnitude. Therefore they reflect a somewhat stronger curve than rolled shapes.

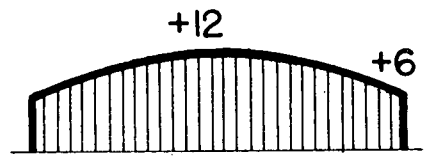
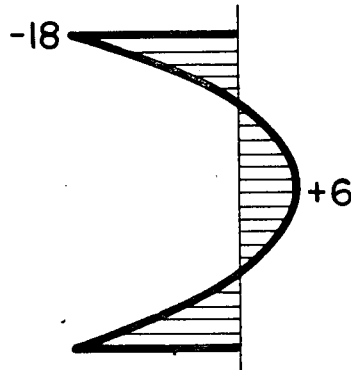
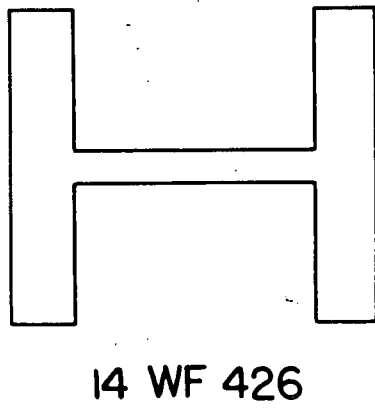
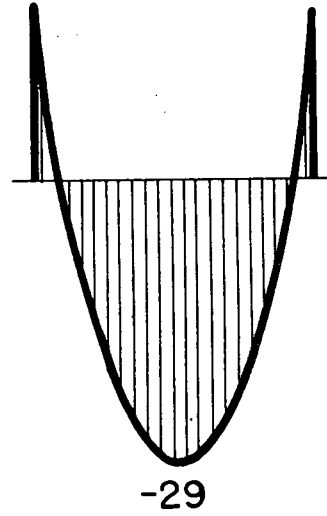
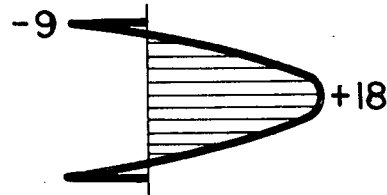
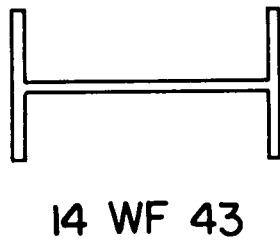
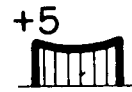
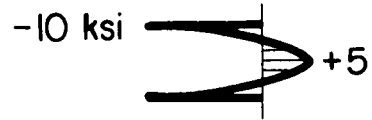
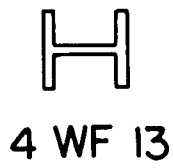
6. The welded columns with higher compressive residual stresses showed a lower strength. ~~The maximum strength~~ The results of further work on the more frequently encountered "box" type of welded column should show a significant increase in strength.

7. ~~The~~ The tangent modulus concept is shown to be the proper basis for determining the maximum strength of centrally-loaded pin-ended steel columns

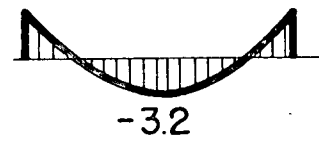
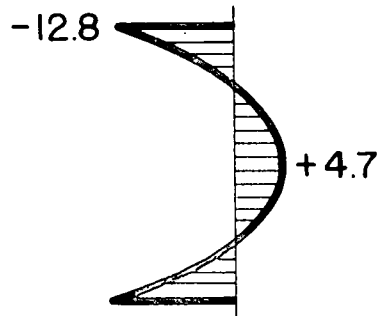


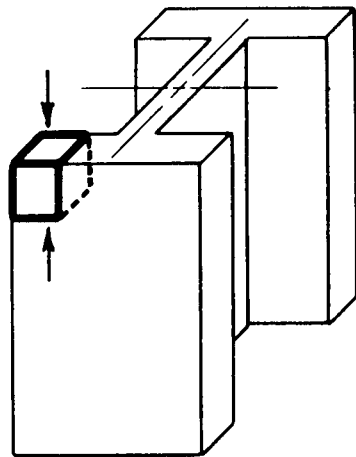
--- Member free of Residual Stress
 — Member Containing Residual Stress



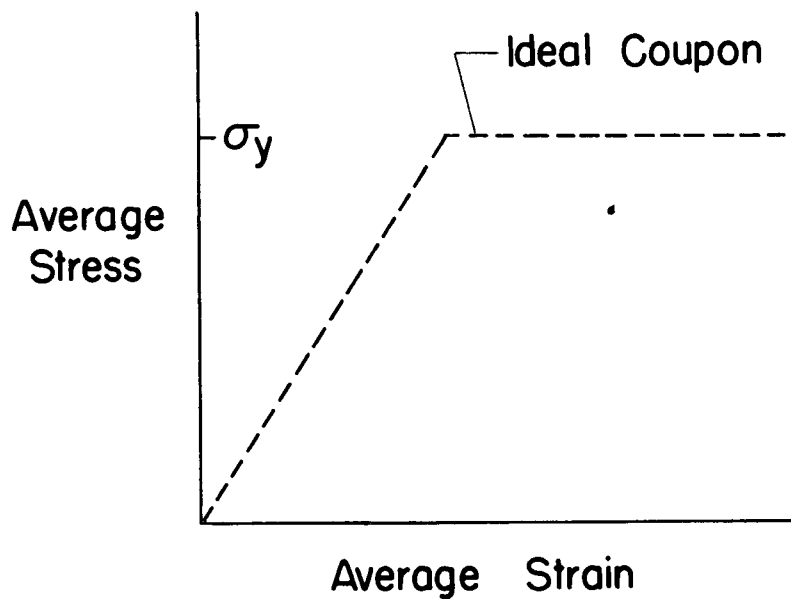


"AVERAGE"
DISTRIBUTION

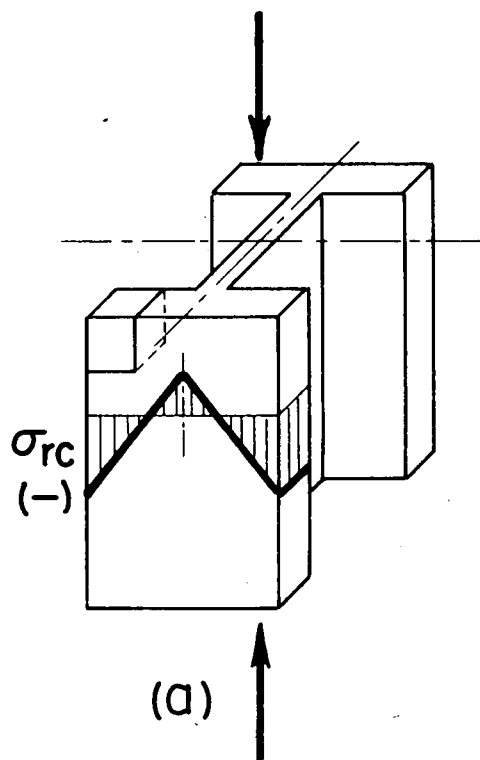




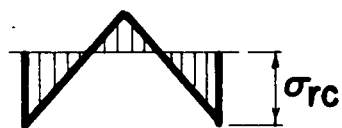
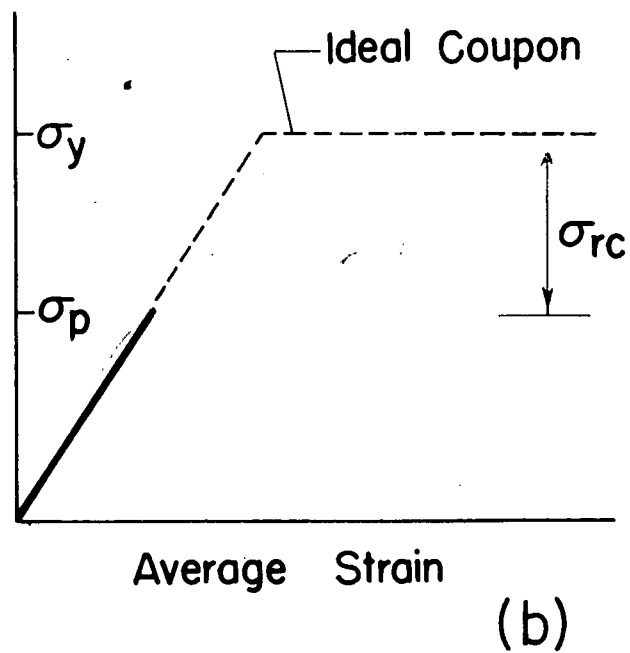
(a)



(b)



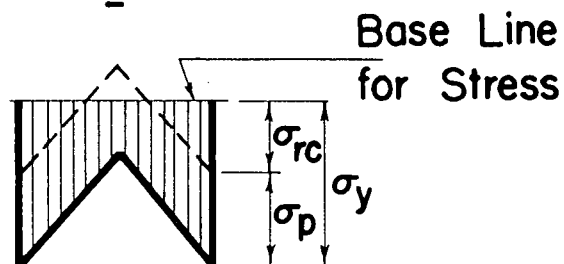
Average
Stress



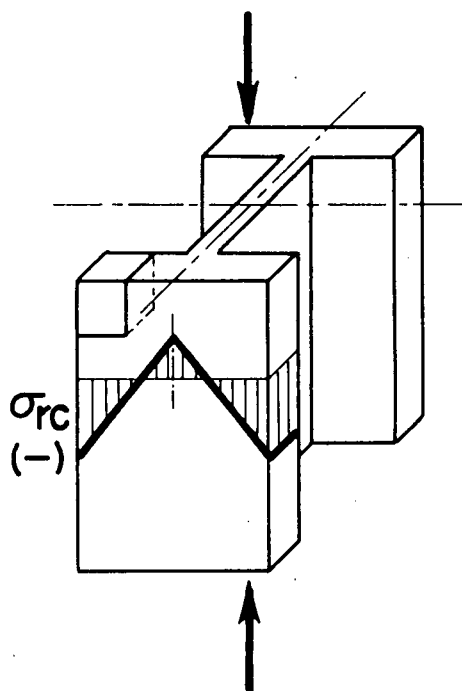
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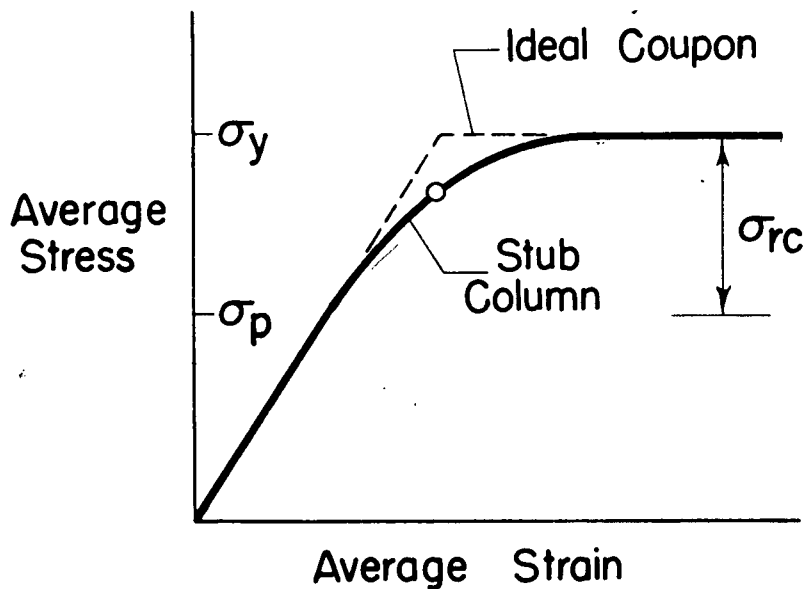
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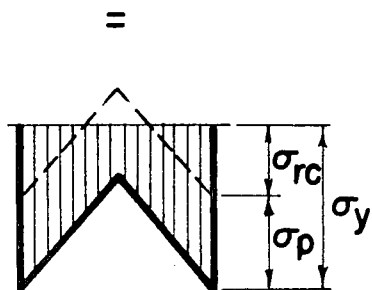
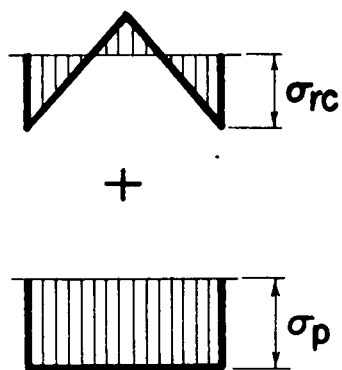
(c)



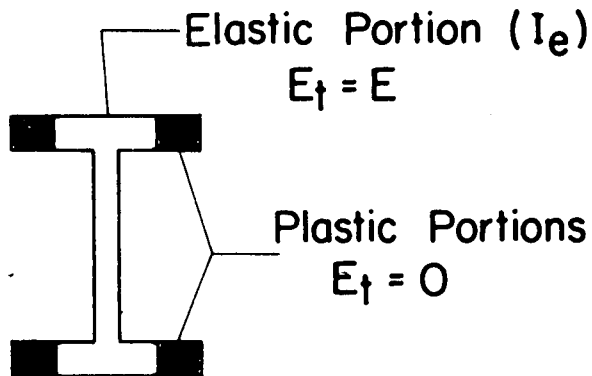
(a)



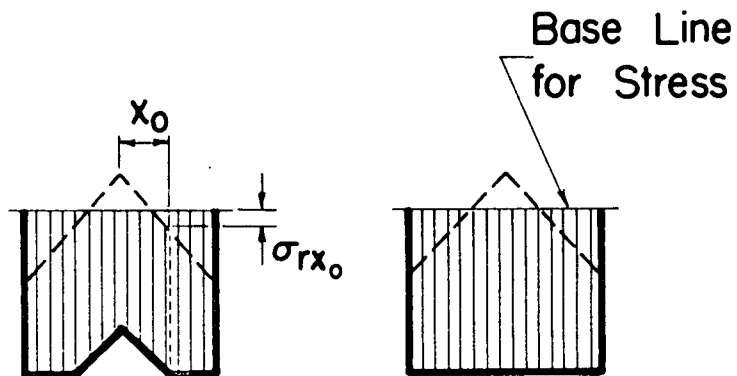
(b)



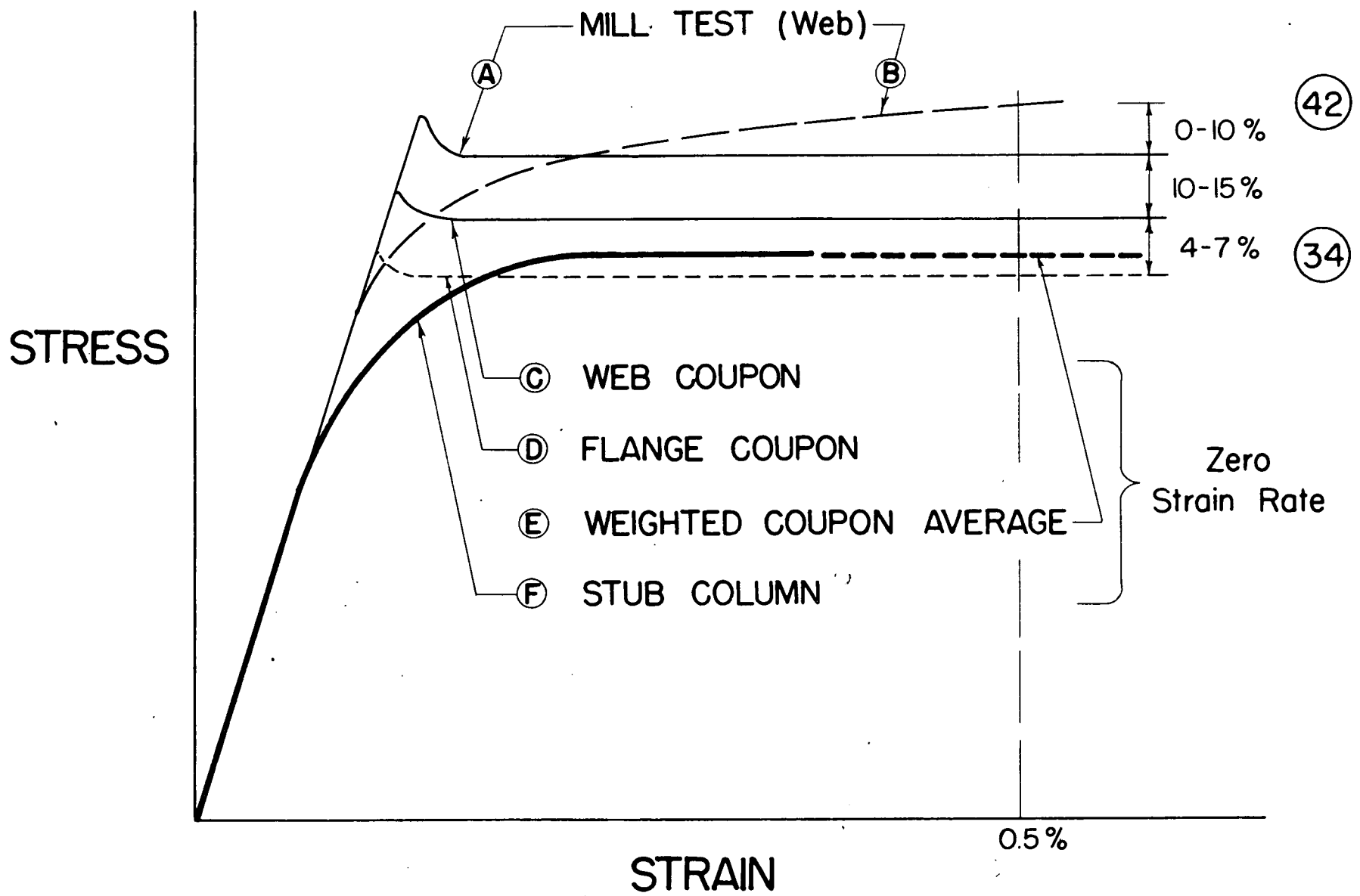
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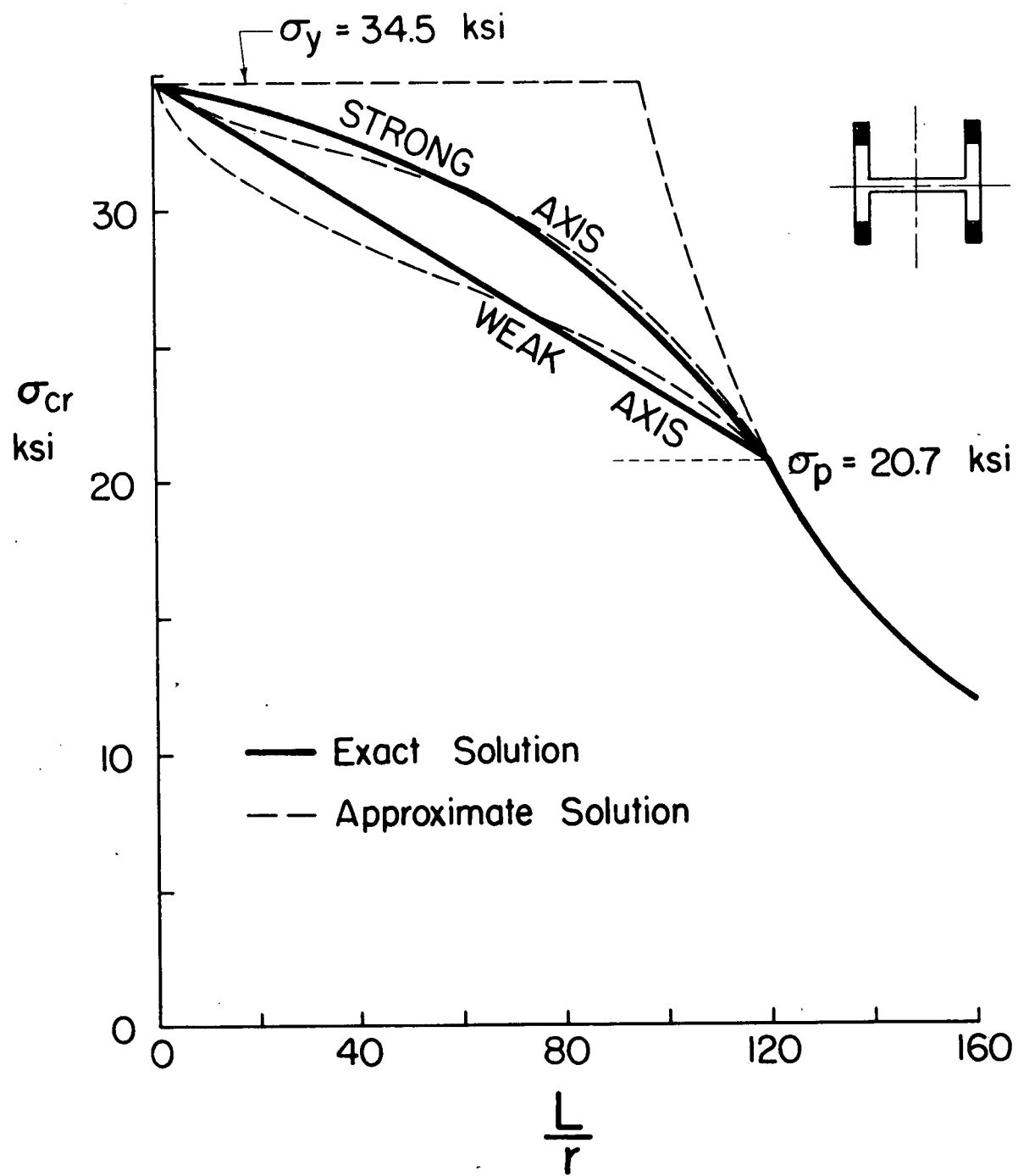


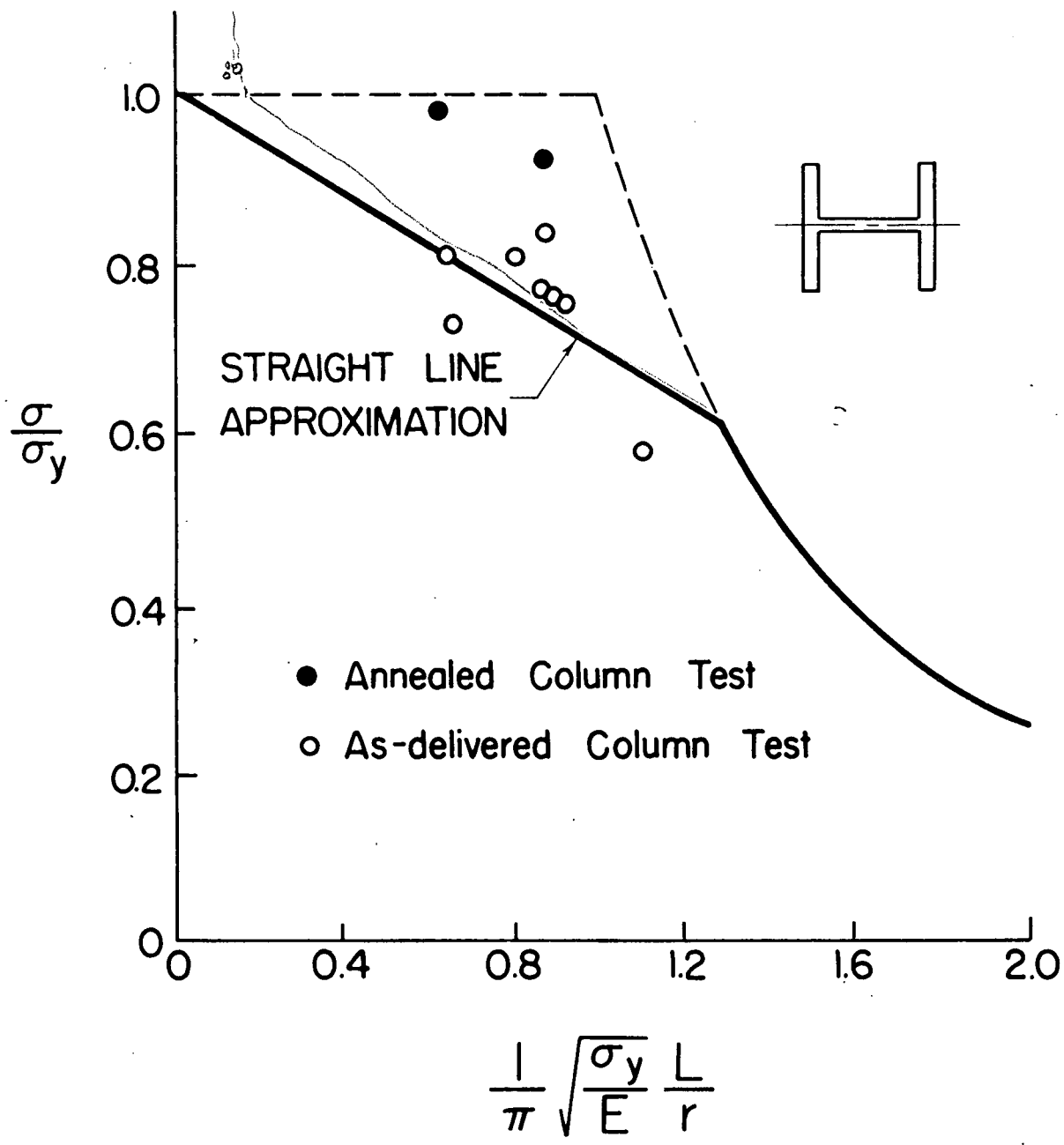
(d)

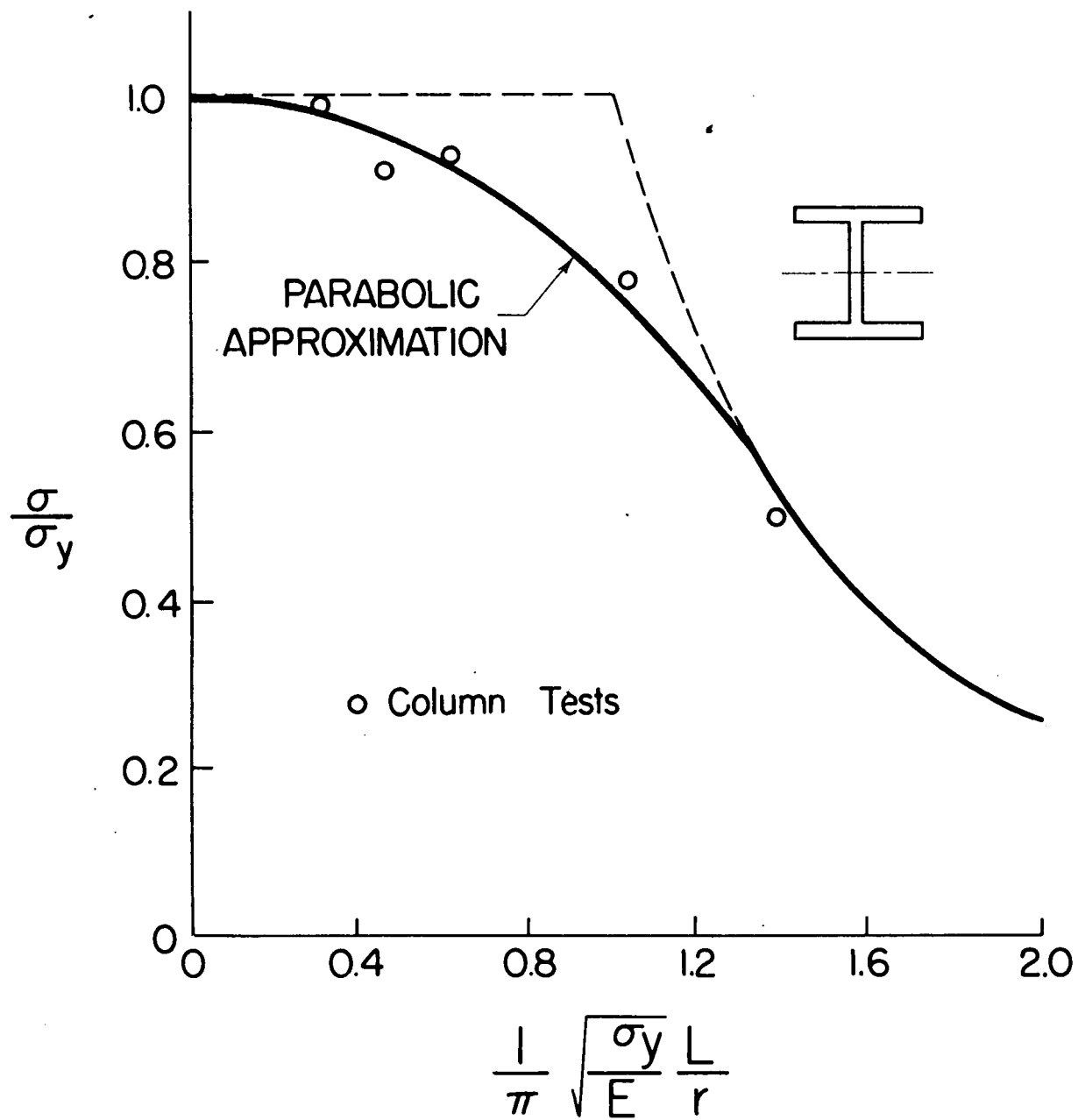


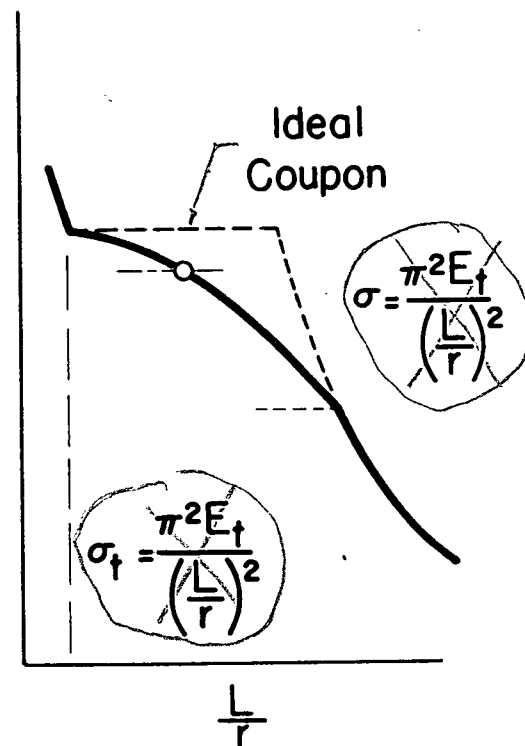
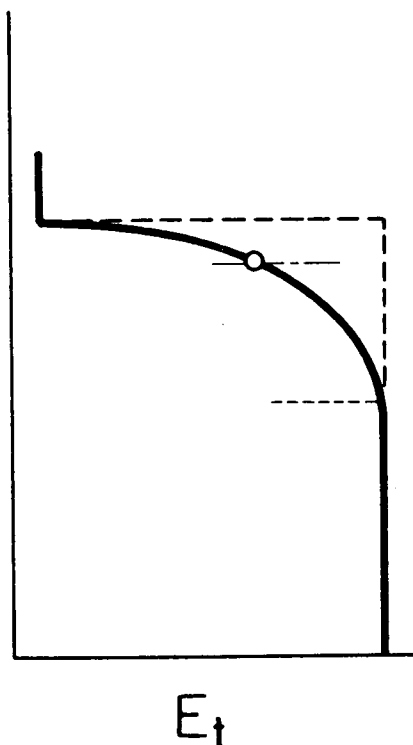
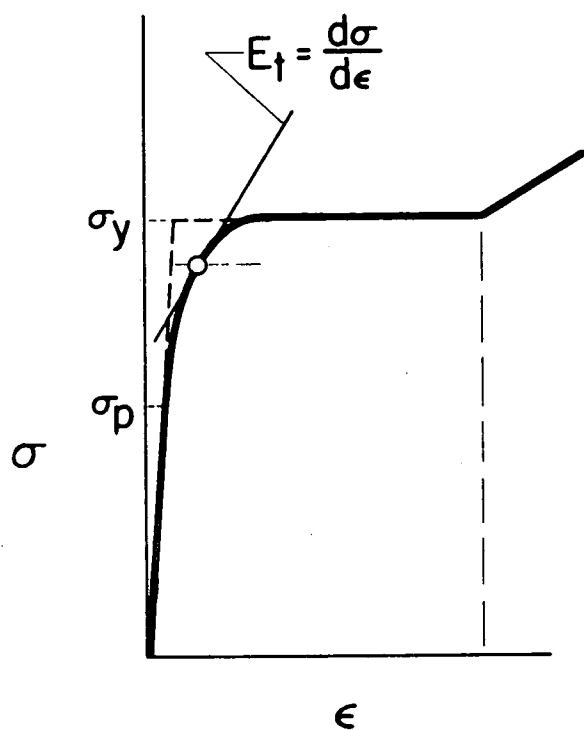
(e)



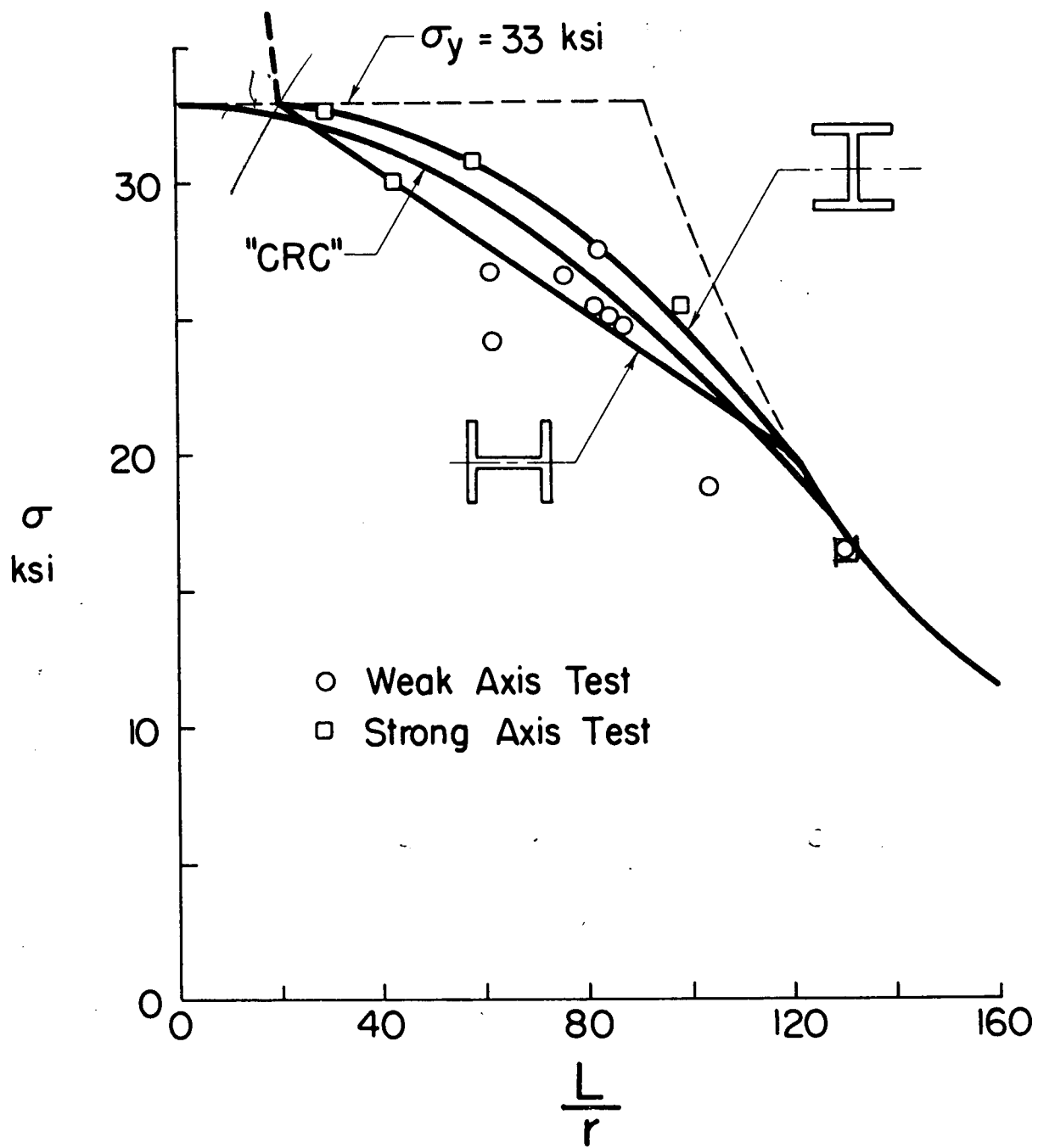


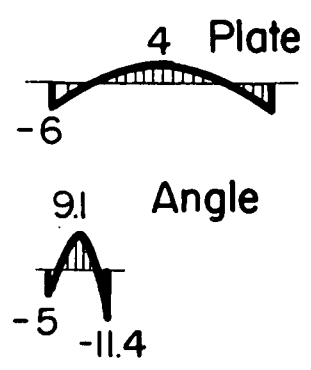
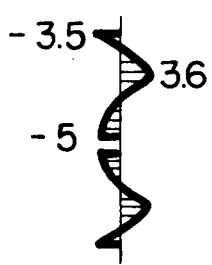
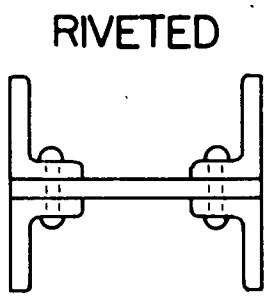
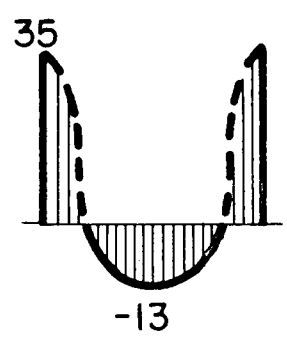
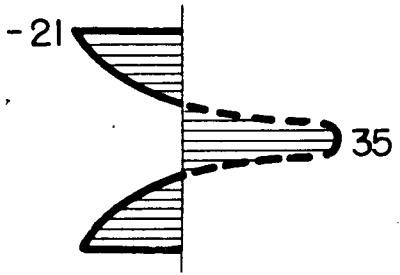
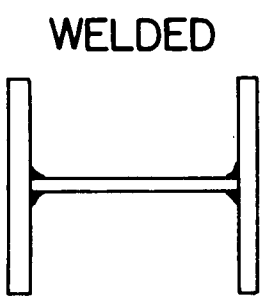
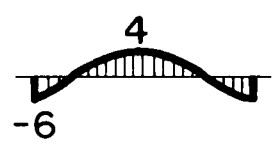
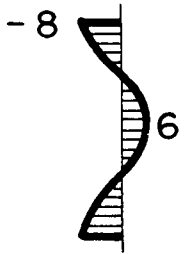
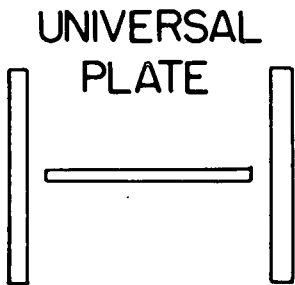
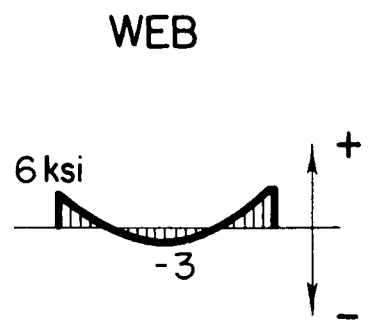
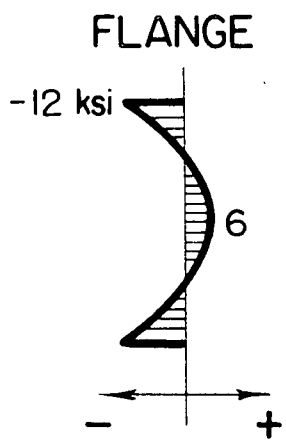
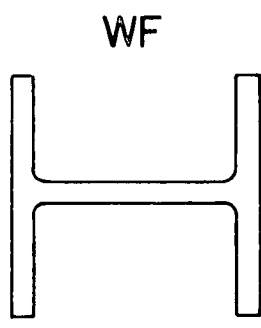


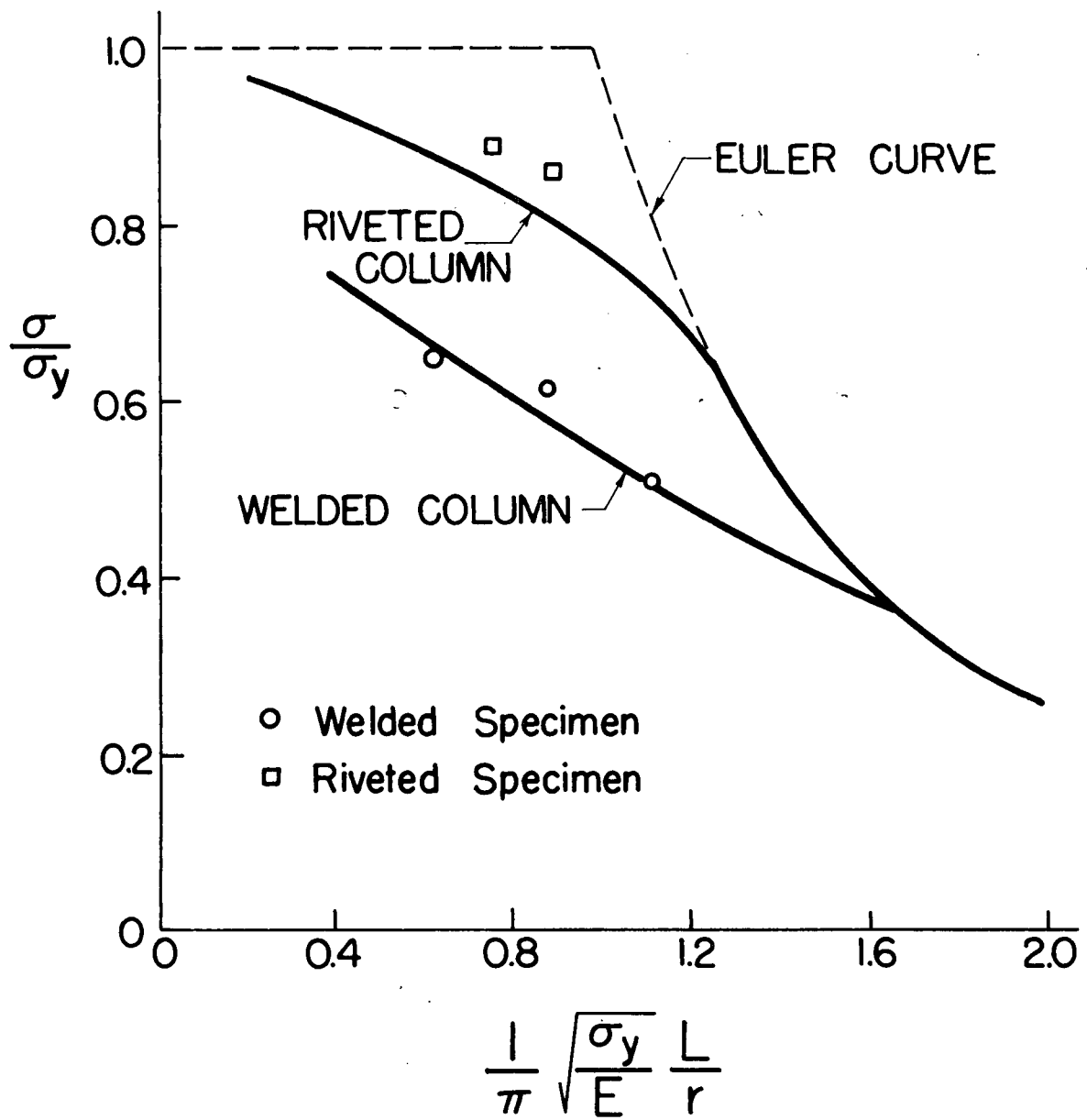


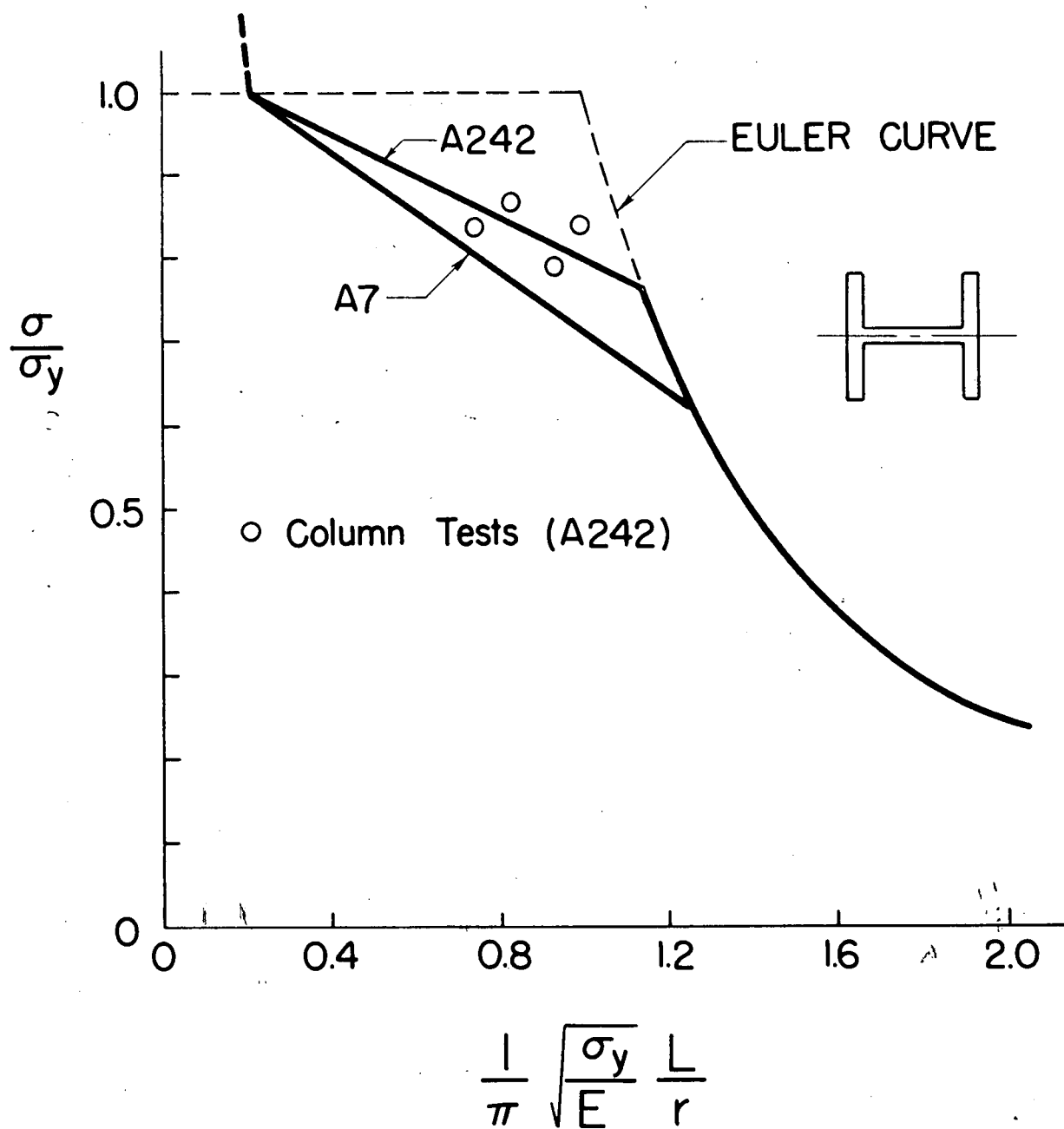


- Member free of Residual Stress
- Member Containing Residual Stress









(b) Note that $x \approx 0.15$ for A242 ($\sigma_y = 55 \text{ ksi}$, $E_{st} = 500 \text{ ksi}$)

